

Determining Hubble's constant Through the Standard Candle Method

Rishabh Jain¹ & Deepak Kumar Choudhary²

¹Podar International School

²Podar International School, Mumbai, Maharashtra.

Abstract: *In this exploration we will be determining the rate of expansion of our known universe (Hubble's Parameter). The method to determine the Hubble's Parameter was to select galaxies in our local cluster and then use an online database to find their respective magnitudes and red shifted or blue shifted velocity of each with respect to earth (taken as a frame of reference¹). Then we determine the relation between recessional velocity and distance with the use of a scatter graphs. The gradient would then give the constant. The correlation coefficient of the graph is found to be 0.191 which indicates a weak positive linear relation between the two variables of distance and recessional velocity impacted.*

A. Introduction

The idea of infinite has always fascinated me. To imagine that we live in a city - in a country - on a continent lying on tectonic plates of our pale blue insignificant dot - near a nuclear reactor with gazillions of its brothers waiting nearby explode in its local cluster of a vast super cluster of an arm in the universe coming out of a singularity: has if not baffled then has blown me! Observing stars in its colossal grandeur provides me with mirth and joy of incomparable amounts. While one must know by now that it is no secret that it is Astronomy where my passion lies, the idea of such cosmic beauty in its vastness has teased me. When my twinkling companions turn out their soothing lights in day, I am left with nothing but to dream and ponder. With their office shut, I decided to investigate more about them and their family like a good old friend. Thus in my exploration I will be determining the rate of expansion of our known universe (Hubble's Parameter). My method to determine the Hubble's Parameter was to select galaxies in our local cluster and then use an online database to find their respective magnitudes and red shifted or blue shifted velocity of each with respect to earth (taken as a

frame of reference²). I would then determine the relation between recessional velocity and distance with the use of a scatter graphs. The gradient would then give me the constant.

My results would be compared to standard theoretically derived value (which I have derived and explained too in Section C).

B.1.1 Preliminary Understanding: The Big Bang Theory

While studying the unit of relativity we learned about the fascinating white holes. Puzzled by the terminology of it, I asked my teacher to reveal more about it - for I had so far known only black holes. To my utmost surprise the answer was trivial and known yet obscure - it was the famed *Big Bang*. The **Big Bang** theory is the prevailing cosmological model for the universe from the earliest known periods through its subsequent large-scale evolution. The model accounts for the fact that the universe expanded from a very high density and high temperature state, and offers a comprehensive explanation for a broad range of observed phenomena, including the abundance of light elements, the cosmic microwave background, large scale structure, and the Hubble's Law. Despite the huge initial expansion, the universe started cooling sufficiently to allow the formation of subatomic particles, and later simple atoms. Giant clouds of these primordial elements later coalesced through gravity to form stars and galaxies

B.1.2 Hubble's Law and His Postulates

In 1929, Carnegie astronomer Edwin Hubble published a linear correlation between the apparent distances to galaxies and their recessional velocities. This simple plot provided evidence that our Universe is in a state of expansion, a discovery that still stands as one of the most profound of the twentieth century¹.

Using photographic data obtained at the 100-in Hooker telescope situated at Mount Wilson, California, Hubble measured the distances to six galaxies in the Local Groupⁱⁱ using the period luminosity relation (discussed in section C.1.1 for Cepheid variablesⁱⁱⁱ). Hubble's construe provided the slope of the velocity versus distance relation yields the Hubble constant, which parameterizes the current expansion rate of the Universe.

The Hubble constant (H_0) is usually expressed in units of kilometers per second per mega parsec (Mpc) and sets the cosmic distance i.e. scale for the present Universe. The inverse of the Hubble constant is the dimensions of time i.e. $T_0 \propto \frac{1}{H_0}$. The Hubble law relates the distance to an object and its redshift: $v = H_0 \times D$ where D is the distance to the object, v is its recessional velocity and H_0 is the constant.

The exact relation between the expansion age and the Hubble constant depends on the nature of the mass energy content of the Universe. In a uniformly expanding universe, the Hubble parameter, H(t), changes as a function of time .

Hubble's law is the name for the observation in physical cosmology that:

-Objects observed in deep space (extragalactic space, ~10 mega parsecs or more) are found to have a Doppler shift interpretable as relative velocity away from the Earth ;

-This Doppler-shift-measured velocity, of various galaxies receding^{iv} from the Earth, is approximately proportional to their distance from the Earth for galaxies up to a few hundred mega parsecs away.

B.1.3: Astronomical Distances

Astronomical distances are vast. For example, it was quoted that the Sun is 25,000 light years (ly) from the center of our galaxy. This of order of magnitude $10^{20} m$. The distance to our nearest star is of the order of magnitude $10^{17} m$. Such vast distances are measured in light years and parsec .A light year is defined as the distance travelled by light in one year. There are about $3.2 \times 10^7 seconds$ in a year and the speed of light is nearly $3 \times 10^8 ms^{-1}$: meaning that the precise value is nearly $9.46 \times 10^{15} m$.

Another useful astronomical distance is that of average distance between the Earth and the Sun, the astronomical unit (AU). $1 AU = 1.5 \times 10^{11} m$.

C: Obtaining the theoretical value.

C.1.1 : Dimension , Shapes And Properties Of Galaxy

The universe is postulated to have the surface of the 4-dimensional ball (called 3-sphere) which is essentially a slice through the universe as a whole for a fixed cosmic time. This slice describes just three spatial dimensions. The observable universe is a small part of this 3-sphere; hence it looks flat (3-dimensional Euclidean space) up to measurement precision (about 0.4% at the moment)ⁱⁱ. It is the addition of time that makes the universe 4-dimensional. The observable part is similar to a 3+1-dimensional Minkowski space-timeⁱⁱ. The universe as a whole may be a de Sitterⁱⁱ space-time. A de Sitter space-time is the analogon of a sphere (i.e. equal to the surface of a ball), in a Minkowski space, instead of an Euclidean space, but it's not literally a sphere. It is postulated to have a critical density of 1 and that it would approach a flat universe in infinite time all in accordance with cosmological principle.

C.1.2 : Value Of The Parameter

The Universe is approximately 92.5 bly in diameter in 2d sphere. Since the universe can be imagined as a 3d volume (Eq.A) in a 4d plane (area defined in Eq.B) : we equate both of them.

$$\frac{4}{3} \times \pi r_0^3 = 2\pi^2 R_H^3 \text{ i.e } A=B \dots \dots \text{Eq.1}$$

where r_0 is the radius of observable universe and R_H is the radius of hyperverses .

$$R_H = \sqrt[3]{\frac{2r_0^3}{3\pi}} = \sqrt[3]{\frac{2}{3\pi}} \times r_0 \dots \dots \text{Eq.2}$$

Since $r_0 \cong 46.25 \text{ bly}$

$$R_H = \sqrt[3]{\frac{2}{3\pi}} \times (46.25 \text{ bly}) \cong 27.5865 \text{ bly}$$

To now , calibrate the rate of expansion (since we assume that the universe started from a singularity the radius pegged is the probable expansion) we simply divide by the presumed age of the universe

$$\text{i.e. } \frac{27.5865 \text{ bly}}{13.8 \text{ billion years}} = 1.999c$$

The universe is thus moving into the 4 dimension at a rate of $\cong 2c$.

The rate at which the circumference is expanding can now be deduced to :

$$\frac{\Delta C_H}{C_H} = \frac{d2\pi \times 2c}{d2\pi R_H} = \frac{2c}{R_H} \text{ where } C_H \text{ is the circumference of the hyperverses}$$

$$\frac{2c}{R_H} = \frac{2 \times 3 \times 10^5 Kms^{-1}}{27.5865 \text{ bly}} = 21749.76891 \frac{kms^{-1}}{bly}$$

Since we are now aware that $1pc = 3.0659 \text{ light year}$ we multiply the whole by 3.0659

$$\frac{21749.76891}{3.0659} = 7094.089 \frac{kms^{-1}}{bpc} = 70.94 \frac{kms^{-1}}{Mpc}$$

D: Obtaining the Experimental Value

D.1.1: Methodology - Standard Candles

A standard candle is a class of astrophysical objects, such as a supernovae or variable stars, which have known luminosity due to characteristic quality possessed by the entire class of objects. Thus, if an extremely distant object can be identified as a standard candle then the absolute magnitudeⁱⁱ M (luminosity) of that object is known. Knowing the absolute magnitude, the distance D (in cm) can be calculated from the apparent magnitude (m) as shown in the formula below.

$$m - M = 5 (\log_{10} d - 1) .$$

The different kinds of "standard Candles", include the Cepheid Variables (star), the planetary nebulae, and the Type Ia supernovae.

D.1.2 : Database Used

For my research I have decided to use the SIMBAD database and the NED database hosted by Caltech and run by NASA . The reason I have decided to use the following databases is due to their high reliability, simplicity to use and comprehend, and alas their regular update feature. To access either of the databases one can simply go their basic search engine and type the messier object or its name to obtain required information.

D.1.2 Selection Of Galaxy For Calibration

There are billions of galaxy in the night sky – each with its characteristic properties and each with something of significant interest to offer. Startled by the sheer numbers I decided to select the most observed and the galaxies at varying distance from ours to get the best and varied absolute magnitude data. Following which I have decided to select : The Andromeda Galaxy (M31) , Triangular Galaxy (M33) , Sombrero Galaxy (M104) , Whirlpool Galaxy (M51) and Bode’s Galaxy (M81)ⁱⁱ

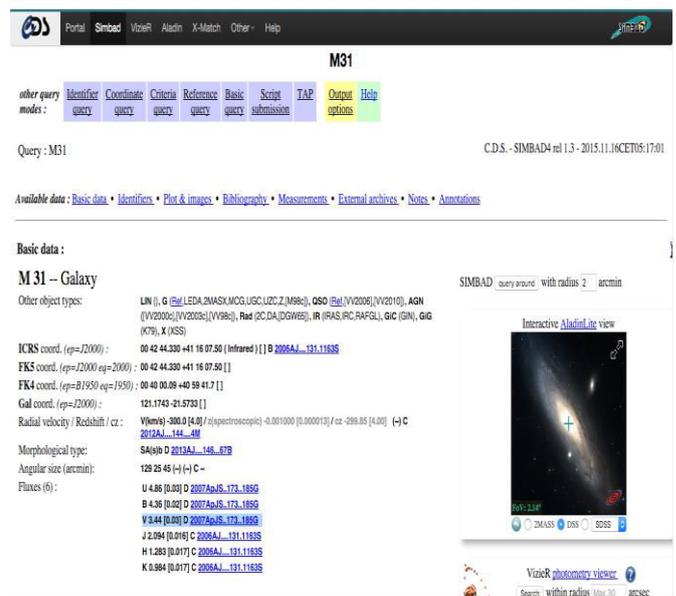


Figure : This is the screenshot of SIMBAD database

D.1.3 Tabulation Of Data

The following table represents the data collected on the variables : Apparentⁱⁱ And Absolute magnitudeⁱⁱ of each galaxy .

Number	Name Of Galaxy	Absolute Magnitude (M)	Apparent Magnitude (m)	Avg(m - M)
1	Andromeda	"-21.02 ±0.092	3.44	24.46 ±0.092
2	Triangulum	"-18.85 ±0.275	5.72	24.57 ±0.275
3	Sombrero	"-20.786 ±0.119	8.98	29.766 ±0.119
4	Whirlpool	"-20.856 ±0.371	8.36	29.216 ±0.371
5	Bode's	"-23.419 ±0.073	6.94	30.359 ±0.073

The difference between the apparent and absolute magnitude is given the distance modulus called ϕ . For example the distance modulus for Andromeda is $3.44 - (- 21.02) = 24.46$

After getting the average ϕ along with the error we put the values in the formula :

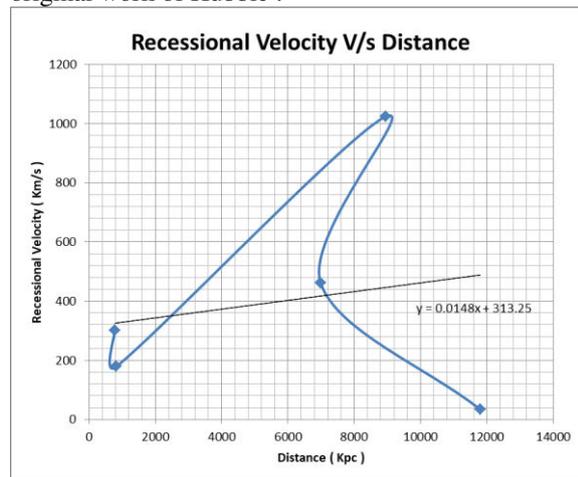
$$\frac{\phi+5}{5} = \log_{10} D \text{ Where } D \text{ is the distance of galaxy from earth .}$$

Number	Name Of Galaxy	ϕ	$(\phi+5)/5$
1	Andromeda	24.46 ± 0.092	5.892 ± 0.018
2	Triangulum	24.57 ± 0.275	5.914 ± 0.055
3	Sombrero	29.76 ± 0.119	6.952 ± 0.024
4	Whirlpool	29.22 ± 0.371	6.844 ± 0.074
5	Bode's	30.36 ± 0.073	7.072 ± 0.014

Taking $\frac{\phi+5}{5}$ as A , the distance $D = 10^A$.

Number	Name Of Galaxy	($\phi \pm 5$)/5	Distance (Kpc)	Recessional Velocity (km/s)
1	Andromeda	5.892 ± 0.018	779.83 ± 32.3305	ⁱⁱ 301 ± 3
2	Triangulum	5.914 ± 0.055	820.35 ± 104.169	ⁱⁱ 179 ± 3
3	Sombrero	6.952 ± 0.024	8953.65 ± 990.97	1024 ± 5
4	Whirlpool	6.844 ± 0.074	6982.32 ± 2390.9	463 ± 3
5	Bode's	7.072 ± 0.014	11809.2 ± 761.11	ⁱⁱ 34 ± 4

After obtaining the distance to each galaxy . We plot a distance V/s recessional velocityⁱⁱⁱ graph as in the original work of Hubble .



Graph Of Recessional Velocity V/s Distanceⁱⁱⁱ

E : Analysis And Data Processing

The gradient of the graph gives us the value of the constant. The derivatives of the function at an individual point are found to be then ranging from 347 $\text{Kms}^{-1}\text{Mpc}^{-1}$ to 3.22 $\text{Kms}^{-1}\text{Mpc}^{-1}$ with the standard gradient being $313.25\text{Kms}^{-1}\text{Mpc}^{-1}$.

Using the equation $y = mx + c$,

The equation which we get for the relationship between the two variables is $y = 0.0148x + 313.25$

The range of parameter, though vast, are compatible with the original Hubble's constant. The correlation coefficient of the graph is found to be 0.191 which indicates a weak positive linear relation between the two variables of distance and recessional velocity impacted. However the coefficient is skewed and does not impact the relation dictated by intensity and distance as I have taken a distinct sample of varied values.

F : Evaluation And Extensions

The constant is varied because of a multitude of factor affecting the data. The absolute magnitude of the galaxy taken in the exploration are affected by the distance factor owing to the presence of interstellar dust, nebulae and gases. This causes restriction in observance of the true luminosity and ultimately magnitude. The data is further impacted by the galactic tide which causes the pulling and attraction of other galaxies which as in the case of Andromeda causes blueshift

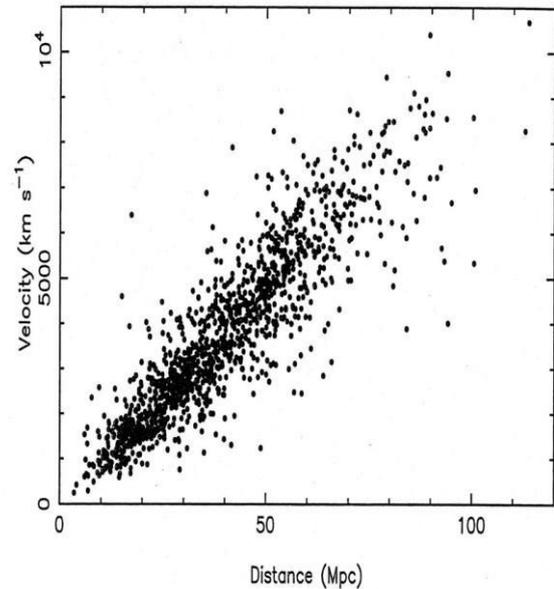


Figure 2.5 A plot of velocity versus estimated distance for a set of 1355 galaxies. A straight-line relation implies Hubble's law. The considerable scatter is due to observational uncertainties and random galaxy motions, but the best-fit line accurately gives Hubble's law. [The x-axis scale assumes a particular value of H_0 .]

ⁱⁱⁱ

i.e. decrease in the wavelength and corresponding frequencies. Since most of the data are taken through spectroscopic filters such as U-V or V-B, this causes a decrease in the apparent magnitude.

The inclinationⁱⁱⁱ of galaxies induces both reddening and extinction to their observed spectral energy distribution, which in turn impact the derived properties of the galaxies. However the effect could be minimized by the accounting of on flux density or on rest frames for the stars/galaxy.

I believe that this exploration could further be extended to calculate the critical density of known universe, study the topological phenomenon of hyperverses, calibrate the brightness and color of stars of galaxy. By the addition of the variable luminosity, the exploration could be furthered chartered deep by the extension of intensity – displacement relationship thereby helping us to predict further cosmological events in this majestic skies of ours.

References

ⁱⁱⁱ A “frame of reference” is just a set of coordinates: something you use to measure the things that matter in Newtonian problems, that is to say, positions and velocities.” : Fowler, Michael. (2008): 3. *Galileo : A Supercomputer For Everyone*. University Of Virginia. Web. 7 Nov. 2015.

<<http://galileo.phys.virginia.edu/classes/252/SpecRelNotes.pdf>>.

ⁱⁱⁱ Hubble, Edwin. "A Relation Between Distance And Radial Velocity Among Extra-Galactic Nebulae by Edwin Hubble." *A Relation Between Distance And Radial Velocity Among Extra-Galactic Nebulae by Edwin Hubble*. National Academy Of Science, n.d. Web. 13 Sept. 2015. <http://apod.nasa.gov/diamond_jubilee/d_1996/hub_1929.html>

ⁱⁱⁱ "This is "our" group of galaxies." - "The Local Group of Galaxies." *The Local Group*. Students For The Exploration And Development Of Space, n.d. Web. 21 Nov. 2015. <<http://messier.seds.org/more/local.html>>.

ⁱⁱⁱ Cepheid Variables Cepheid Variables are the most important type of variable because it has been discovered that their periods of variability are related to their absolute luminosity. This makes them invaluable as a contributor to astronomical distance measurement. Refer to <http://hyperphysics.phy-astr.gsu.edu/hbase/astro/cepheid.html>

ⁱⁱⁱ Hubble, Edwin. *Astronomy E.Hubble* (n.d.): 168-73. Proceedings of the National Academy of Sciences of the United States of America PNAS, Proceedings of the National Academy of Sciences PNAS, Proceedings of the National Academy of Sciences. Web. 25 Aug. 2015. <<http://www.pnas.org/content/15/3/168.full.pdf>>.

ⁱⁱⁱ The velocity of a galaxy is measured by the Doppler effect, the fact that light emitted from a source is shifted in wavelength by the motion of the source. The change in wavelength, with respect to the source at rest, is called the redshift (if moving away, blueshift if moving towards the observer) and is denoted by the letter z. Redshift, z, is proportional to the velocity of the galaxy divided by the speed of light. Since all galaxies display a redshift, i.e. moving away from us, this is referred to as recession velocity. - Schombert, James. "Expanding Universe." *Expanding Universe*. University of Oregon, n.d. Web. 18 Oct. 2015. <<http://abyss.uoregon.edu/~js/cosmo/lectures/lec14.html>>.

ⁱⁱⁱ "Will the Universe Expand Forever?" *WMAP-Shape of the Universe*. Wilkinson Microwave Anisotropy Probe, n.d. Web. 17 Nov. 2015. <http://map.gsfc.nasa.gov/universe/uni_shape.html>.

ⁱⁱⁱ Poincare, Henri. "Minkowski, Mathematicians and the Mathematical Theory of Relativity." *Minkowski, Mathematicians and the Mathematical Theory of Relativity*. Le Laboratoire D'Histoire Des Sciences Et De Philosophie - Archives Henri Poincaré, n.d. Web. 21 Nov. 2015. <<http://henripoincarepapers.univ-lorraine.fr/walter/papers/1999-mmm-walter.html>>.

ⁱⁱⁱ "The de Sitter metric describes an exponentially expanding universe. Our universe

seems to be asymptotically de Sitter at very early and very late times." : Anninos, Dionysios. "De Sitter Gauge Theory." *Gravitation and Gauge Symmetries* (2002): n. pag. <Http://arxiv.org/pdf/1205.3855.pdf>. Stanford Institute Of Theoretical Physics. Web. Oct.-Nov. 2015. <<http://arxiv.org/pdf/1205.3855.pdf>>.

ⁱⁱⁱ In 1929, Hubble estimated the value of the expansion factor, now called the Hubble constant, to be about 500 km/sec/Mpc. Today the value is still rather uncertain, but is generally believed to be in the range of 45-90 km/sec/Mpc. While in general galaxies follow the smooth expansion, the more distant ones moving faster away from us, other motions cause slight deviations from the line predicted by Hubble's Law. : "Hubble's Law." *Hubble's Law*. Cornell University, n.d. Web. 14 Nov. 2015.

<http://www.astro.cornell.edu/academics/courses/astro201/hubbles_law.htm>.

ⁱⁱⁱ "The units used to describe brightness of astronomical objects. The smaller the numerical value, the brighter the object." : "International Comet Quarterly." *Glossary of Astronomical Terms*. Department Of Earth And Planetary Sciences - Harvard University, n.d. Web. 10 Nov.2015.

<<http://www.icq.eps.harvard.edu/ICQGlossary.html#Magn>>.

ⁱⁱⁱ The following galaxies had been selected also on the basis of data availability and their distinctiveness .

ⁱⁱⁱ Denoted by the alphabet v in the database .

ⁱⁱⁱ Denoted by the alphabet M in the database .

ⁱⁱⁱ Recessional velocity is the rate at which an object is moving away, typically from Earth.

ⁱⁱⁱ The error bars of the graphs are not visible due to their relatively small size compared to the compatible distances .

ⁱⁱⁱ Note that the graph appears to contact primarily because of the difference in the units of the independent variable i.e. the x-axis . However for processing my values I have indeed reverted to Mpc instead of Kpc as depicted in my graph. Also the following image has been taken from : N.p., n.d. Web. 22 Nov. 2015. <<http://astro.uni-wuppertal.de/~kampert/Cosmology-WS0607.html>>.

ⁱⁱⁱ **Orbital inclination/Angular Inclination** is the [angle](#) between a [reference plane](#) and the [orbital plane](#) or [axis](#) of direction of an object in [orbit](#) around another object

