Imperial Journal of Interdisciplinary Research (IJIR)
Vol-3, Issue-8, 2017
ISSN: 2454-1362, http://www.onlinejournal.in

The Effects of Some Physical Parameters on Ground Temperature with Time-Dependent Suction Velocity in the Presence of Internal Heat Generation

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Abstract: *This research investigated the effects of some physical parameters on ground temperature with a vegetative cover in an infinitely horizontal direction with time-dependent suction velocity in the presence of internal heat generation. The governing equations (continuity equation and energy equation) were reduced to non-dimensional form by the use of some dimensionless parameters resulting into a set of partial differential equations. These equations were then solved analytically using perturbation method and the numerical results obtained using MATLAB 7.9.0 (R2009b) software. The effects of governing physical parameters, such as Radiation parameter (R), Prandlt number (Pr) and frequency of oscillation (ω) were examined on the resulted temperature and Biot number. The results were displayed on tables and graphs, and were compared with existing works in literature. The result obtained is very crucial in addressing problems associated with ground temperature especially when the depth considered is profound and the internal heat generation is very significant.*

Keywords: *Physical Parameters, ground temperature, suction velocity, internal heat generation*

1. Introduction

Internal heat generation is the heat that is been generated within the ground (or earth's interior). Much of the Earth's internal heat is created by decay of naturally radioactive elements. An estimated 45 to 90 percent of the heat escaping from the Earth originates from radioactive decay of elements mainly located in the mantle [9]. Other sources may include heat of impact and compression released during the original formation of the Earth by accretion of in-falling meteorites, abundant heavy metals (iron, nickel, copper) descended to the Earth's core, latent heat released as the liquid outer core crystallizes at the inner core boundary, heat generated by tidal force on the Earth as it rotates; since rock cannot flow as readily as water it compresses and distorts, generating heat [11]. This is an object of concern, for example, in mining process where Deep-level mining are becoming more prevalent due to the depletion of ore bodies closer to the surface – many of the easily accessible ore bodies have already been found and mined [6]. In addition, deeper mining is becoming more common in existing mines as the shallower parts of the ore bodies have been exhausted. However, the safety of the miners becomes a concern due to increase in the ground temperature as the mining gets deeper [6].

Information about ground temperature is also important in agriculture, architectural designs and in engineering, which all put into reasonable considerations the time and depth of the ground.

Ground temperature varies due to changes in radiant, thermal and latent exchange processes taking place through the soil surface. The effect is propagated into the soil profile by transport processes which are affected by space and time variable along with some thermo-physical soil properties like specific heat capacity, thermal conductivity and thermal diffusivity [4]. At the surface of the earth, energy is dominated by the incoming solar radiation which is about 173,000 terawatt (TW) [3]. However, Heat flows constantly from its sources within the Earth to the surface. Estimates of the total heat flow from Earth's interior to surface span range of 43 to 49 terawatt (TW). The closest estimate is 47 TW, an average crust heat flow of 91.6 mW/m^2, and is based on more than 38,000 measurements. The respective mean heat flows of continental and oceanic crust are 70.9 and 105.4 mW/m^2 [5].

Samara *et al.* [7] studied a comparison between the temperatures at different soil depths and atmospheric physical factors such as air temperature, humidity, precipitation, wind speed and solar radiation, in Keller Peninsula, located in King George Island, Antarctica Maritime. Their study revealed that the increase in ice-free areas in the Antarctic, as a result of climate change, caused a series of local transformations that may affect the microclimate of the soil, causing greater melting of the permafrost and causing changes in the soil moisture. Cyril and Difference [2] investigated Temperature Variation at Soil Depths in Parts of Southern Nigeria. They observed that the soil temperature varies randomly as a result of considerable different characteristics possessed by different soil types. Uno *et al* [10] studied parametric analysis of ground temperature profile in Bwari-North Central Nigeria. They found out that there had been striking features of the ground temperatures over five years which shows that even when natural forces (rainfall e.t.c) tried balancing the net earth radiation, the reality of climatic change steps in. Since there is an unabated solar radiation increase in the region, what might likely happen in the future is a function of the subsequent data's and the precautionary measures put in place by the government to facilitate the ozone UV healing.

In this research work, effort was made to present a mathematical model on ground temperature which considered effects of some physical parameters such as the radiation, prandlt number and frequency of oscillation in the presence of internal heat generation, in an isotropic condition and an optically thin environment.

2. Mathematical Formulation

We consider effects of some physical parameters on ground temperature in an infinitely horizontal direction in the presence of internal heat generation. The flow is two-dimensional and considered to be infinitely in the horizontal direction ($y-axis$). The x and y axes are decomposed to form the horizontal axis ($y-axis$), with the vertical axis ($z-axis$) taken to be in the soil. Radiation is considered to be from an external source in a direction due to gravity. The ground is taken to be an optically thin environment where thermal conductivity of the soil and other soil properties are assumed to be constant. Under the above assumptions and before the introduction of dimensionless variables, the dimensional governing equations are as follows:

Continuity equation

$$\frac{\partial w'}{\partial z'} = 0 \tag{1}$$

Energy equation

The ground is assumed to be an optically thin environment and by Tarkher *et al.* [8], radiative heat flux takes the form:

$$\frac{\partial q_r'}{\partial z'} = 4\alpha^2 \left(T_w' - T_\infty'\right) \tag{2}$$

Then the energy equation became:

$$\frac{\partial T'}{\partial t'} + \frac{w'\partial T'}{\partial z'} = \frac{1}{\rho C_p}\left\{k\frac{\partial^2 T'}{\partial z'^2} - 4\alpha^2\left(T_w' - T_\infty'\right)\right\} + \frac{Q_0}{\rho C_p}\left(T' - T_\infty'\right) \tag{3}$$

subject to:

$T'(0,t) = T_w'$ on $z = 0$ (Temperature of the ground surface)

$$T'(z,t) = T_\infty' \text{ as } z \to \infty \tag{4}$$

where z' is the dimensional depth of the ground (the distance perpendicular to y'), t' is the dimensional time, w' is the components of dimensional velocities along z' direction, T' is the dimensional temperature, T_∞' is the free stream dimensional temperature, T_w' is the dimensional wall temperature, ρ is the density, C_p is the specific heat capacity, k is the thermal conductivity, α is the absorption coefficient and q_r' is the Radiative heat flux.

In order to write these governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities were introduced:

$$t = \frac{w_0'^2 t'}{v}$$

$$\Rightarrow \quad \frac{\partial}{\partial t'} = \frac{\partial}{\partial t}\cdot\frac{\partial}{\partial t'}\left(\frac{w_0'^2 t'}{v}\right) = \left(\frac{w_0'^2}{v}\right)\cdot\frac{\partial}{\partial t}$$

$$\theta = \frac{T' - T_\infty'}{T_w' - T_\infty'}$$

$$\Rightarrow T' - T_\infty' = \theta\left(T_w' - T_\infty'\right) \quad \Rightarrow T' = \theta\left(T_w' - T_\infty'\right) + T_\infty'$$

$$z = \frac{w_0' z'}{v}$$

$$\Rightarrow \quad \frac{\partial}{\partial z'} = \frac{\partial}{\partial z}\cdot\frac{\partial}{\partial z'}\left(\frac{w_0' z'}{v}\right) = \frac{w_0'}{v}\cdot\frac{\partial}{\partial z}$$

$$\omega = \frac{v}{w_0'}\omega' \tag{5}$$

Also, from the continuity equation, w' is not a function of z', so it is either a constant or a function of time.
Thus,

$w' = -w_0'$ or $w' = -w_0'\left(1 + \varepsilon A e^{i\omega' t}\right)$

Following Akinpelu *et al* [1] and for the purpose of this study, w' is taken to be:

$$w' = -w_0'\left(1 + \varepsilon A e^{i\omega' t}\right) \tag{6}$$

That is, a varying suction velocity. Where, w_0' is the initial suction velocity, ω is the frequency of oscillation, and A is the suction parameter. The negative sign is an indication that the suction is towards the ground surface. A and $\varepsilon(epsilon)$ are very small parameters such that $\varepsilon A << 1$.

The energy equation in non-dimensional form then becomes:

$$P_r \frac{\partial \theta}{\partial t} - P_r\left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial z^2} - R^2 + Q\theta \tag{7}$$

where,

$P_r = \frac{v\rho C_p}{k}$ (the Prandlt number)

$R = \sqrt{\frac{4v^2\alpha^2}{k\, w_0'^2}}$ (the Radiation parameter)

$Q = \frac{Q_0\, v^2}{k\, w_0'^2}$ (the heat generation parameter)

subject to:

$\theta = 1$ on $z = 0$ (Temperature at the surface)

$\theta \to 0$ as $z \to \infty$ (8)

3. Method of Solution

Introducing the regular perturbation method, the solution was assumed to be of the form:

$$\theta(z,t) = \theta_0(z) + \theta_1(z)\varepsilon e^{i\omega t} + \cdots \tag{9}$$

Now, taking orders of ε from the energy equation, but neglecting higher order terms $O(\varepsilon^2)$, it yielded the following:

$$\frac{\partial^2 \theta_0}{\partial z^2} + P_r \frac{\partial \theta_0}{\partial z} + Q\theta_0 = R^2 \tag{10}$$

$$\frac{\partial^2 \theta_1}{\partial z^2} + P_r \frac{\partial \theta_1}{\partial z} + (Q - P_r i\omega)\theta_1 = -P_r A c_2 m_2 e^{m_2 z} \tag{11}$$

Using the assumed solution, the corresponding boundary conditions (8) could be rewritten as:

$\theta_0 = 1,\ \theta_1 = 0$ on $z = 0$

$\theta_0 \to 0,\ \theta_1 \to 0$ as $z \to \infty$ (12)

Applying the boundary conditions (12), the equations became:

$$\theta_0 = c_2 e^{m_2 z} + \frac{R^2}{Q} \tag{13}$$

$$\theta_1 = c_6\left(e^{m_2 z} - e^{m_4 z}\right) \tag{14}$$

Thus, based on the assumed solution (9), the solution of the energy equation was derived to be:

$$\theta(z,t) = c_2 e^{m_2 z} + \frac{R^2}{Q} + c_6\left(e^{m_2 z} - e^{m_4 z}\right)\varepsilon\, e^{i\omega t} \tag{15}$$

where,

$$m_2 = -\left(\frac{P_r}{2} + \sqrt{\frac{P_r^2}{4} - Q}\right)$$

$$m_4 = -\left(\frac{P_r}{2} + \sqrt{\frac{P_r^2}{4} + P_r i\omega - Q}\right)$$

$$c_2 = 1 - \frac{R^2}{Q}$$

$$c_6 = \frac{-P_r A c_2 m_2}{m_2^2 + P_r m_2 + Q - P_r i\omega}$$

3.1 The Biot Number

By the temperature field, the Biot number at the ground surface can be determined as follows:

$$B_i = -\left(\frac{\partial \theta}{\partial z}\right)_{z=0} \tag{16}$$

It then resulted to

$$B_i = -c_2 m_2 - c_6(m_2 - m_4)\varepsilon\, e^{i\omega t} \tag{17}$$

4. Results and Discussion

4.1 Analysis of Results

In this section, the numerical representation of the expressions for the temperature and Biot number obtained in the previous section were displayed on tables and graphs to better illustrate the behaviour of the physical quantities used. In order to do this, numerical values of these quantities were computed with respect to the variations in the governing parameters, namely, the

Radiation parameter (R), the Prandtl number (Pr) and the frequency of oscillation (ω). In the study, the following default parametric values were adopted:

$R = 0.5$, $\Pr = 0.71$, $Q = 0.1$, $\omega = \pi/2$, $\varepsilon = 0.01$, $t = 1.0$, and $A = 0.5$.

The depth of the ground (z) is measured in metres. All the tables and Figures therefore correspond to these values except otherwise stated.

Table 1. Numerical values of the Temperature (θ) at different values of Radiation parameter (R)

Ground depth (z)	Temperature (θ) at R=0.4	Temperature (θ) at R=0.5	Temperature (θ) at R=0.6	Temperature (θ) at R=0.7
0.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.2423	1.6058	2.0501	2.5751
2.0000	1.3867	1.9668	2.6757	3.5136
3.0000	1.4728	2.1819	3.0486	4.0729
4.0000	1.5241	2.3102	3.2710	4.4064
5.0000	1.5547	2.3867	3.4036	4.6055
6.0000	1.5730	2.4324	3.4828	4.7242
7.0000	1.5839	2.4597	3.5301	4.7951
8.0000	1.5904	2.4759	3.5583	4.8374
9.0000	1.5943	2.4856	3.5751	4.8627
10.0000	1.5966	2.4914	3.5851	4.8777

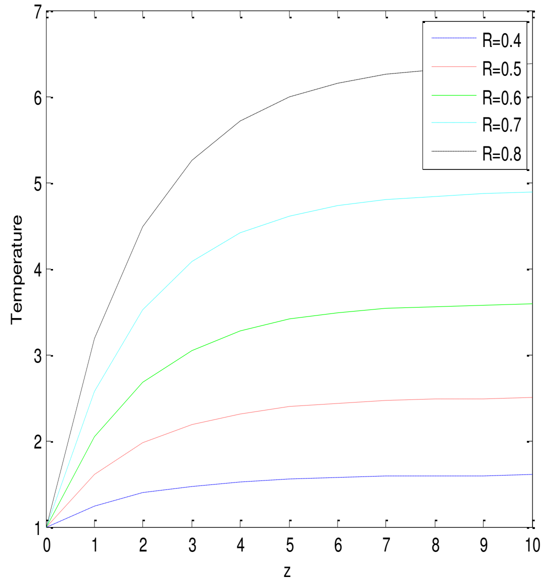
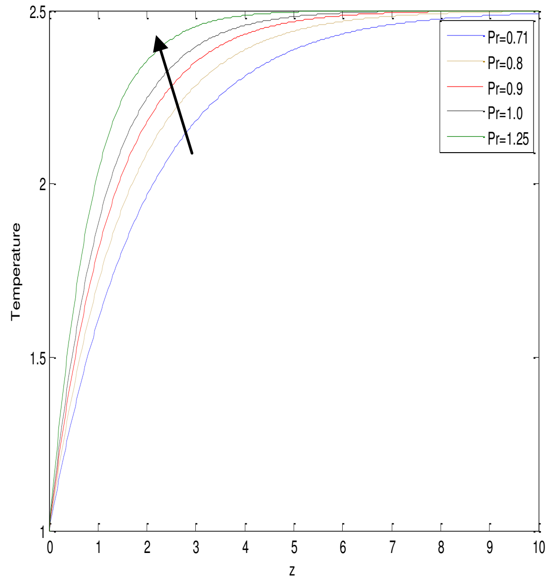


Figure 1. Temperature profile for $\Pr = 0.71,\ Q = 0.1,\ \omega = \frac{\pi}{2},\ \varepsilon = 0.01,\ t = 1.0,\ A = 0.5$ at different values of Radiation parameter (R).

Table 2. Numerical values of the Temperature (θ) at different values of prandtl number (Pr)

Ground depth (z)	Temperature (θ)at Pr=0.71	Temperature (θ) at Pr=0.8	Temperature (θ) at Pr=0.9	Temperature (θ) at Pr=1.0
0.0000	1.0000	1.0000	1.0000	1.0000
1.0000	1.6058	1.7139	1.8066	1.8834
2.0000	1.9668	2.0879	2.1793	2.2464
3.0000	2.1819	2.2838	2.3516	2.3956
4.0000	2.3102	2.3866	2.4313	2.4570
5.0000	2.3867	2.4405	2.4682	2.4823
6.0000	2.4324	2.4688	2.4853	2.4927
7.0000	2.4597	2.4836	2.4932	2.4970
8.0000	2.4759	2.4914	2.4968	2.4988
9.0000	2.4856	2.4955	2.4985	2.4995
10.0000	2.4914	2.4976	2.4993	2.4998

Figure 2. Temperature profile for $R = 0.5,\ Q = 0.1,\ \omega = \frac{\pi}{2},\ \varepsilon = 0.01,\ t = 1.0,\ A = 0.5$ at different values of prandtl number (Pr).



Upward movement of the graph →

Figure 3. Temperature profile for $\Pr = 0.71, R = 0.5,\ Q = 0.1,\ \varepsilon = 0.01,\ A = 0.5,\ z = 0.01$ at different frequency of oscillation (ω).

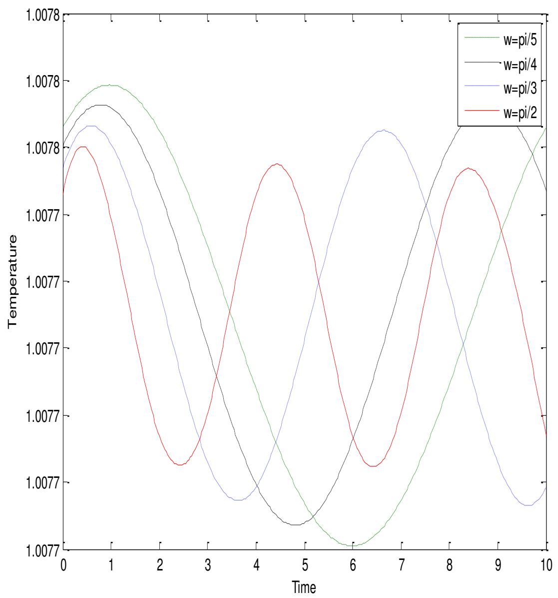


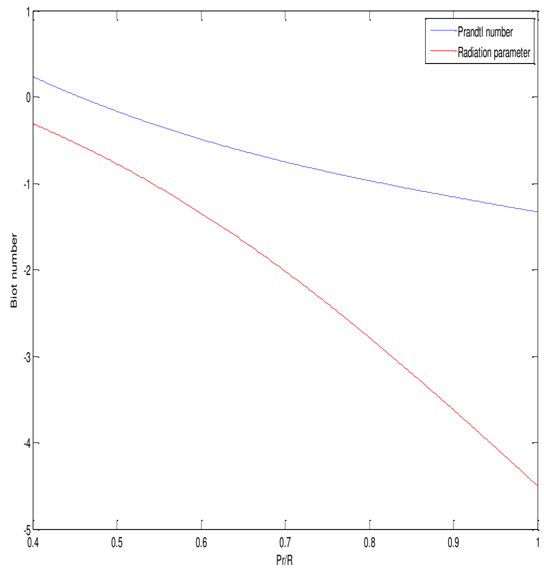
Table 3. Numerical values of effect of Radiation (R) on Biot Number (B_i).

Radiation parameter (R)	Biot number (B_i)
0.4000	-0.3104
0.5000	-0.7759
0.6000	-1.3450
0.7000	-2.0175

Table 4. Numerical values of effect of Prandtl number (Pr) on Biot Number (B_i) .

Prandtl number (Pr)	Biot number (B_i)
0.1700	-0.7759
0.8000	-0.9692
0.9000	-1.1573
1.0000	-1.3332

Figure 4. Biot number (B_i) profile against different values of Prandtl number (Pr) and Radiation parameter (R) with $Q = 0.1,\ \omega = \frac{\pi}{2},\ \varepsilon = 0.01,\ t = 1.0,\ A = 0.5$



4.2 Discussion of Results

Table 1 and Figure 1 both present the typical temperature profile for various values of the Radiation parameter (R) with increase in depth of the ground. It was observed that an increase in the radiation parameter (R) resulted in an increase in the ground temperature. Table 2 and Figure 2 show the temperature profile for various values of the Prandtl number (Pr). It was discovered that an increase in the Prandtl number increased the temperature as well. Figure 3 depicts the sinusoidal variation of temperature with time at each frequency of oscillation (ω) of the suction velocity. As the frequency of oscillation (ω) increased, so the amplitude of the oscillation decreased. This is apparent because temperature is time dependent. Tables 3 - 4 and Figure 4 show that an increase in the Prandtl number and the Radiation parameter decreased the Biot number.

5. Conclusion

The research investigated the effects of some physical parameters on the ground temperature under a vegetative cover in an infinitely horizontal direction in the presence of internal heat generation. The governing equation was reduced to non-dimensional form by the use of some dimensionless parameters resulting into a partial differential equation. The solution for the transient temperature was obtained analytically using perturbation method and numerical results were obtained using MATLAB 7.9.0 (R2009b) software. It was discovered that an increase in radiation parameter and Prandtl number as the ground goes deeper to where the internal heat generation is much significant led to increase in temperature distribution. This is where for example, the safety of the miners in Deep-level mining calls for an attention. However, increase in both Radiation parameter (R) and Prandtl number (Pr) decreased the Biot number. Moreover, there is a sinusoidal variation in ground temperature with time at each frequency of oscillation of the suction velocity.

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