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Robust Nonlinear Geometric Angle Tracker for Aero Pendulum

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***Abstract**: Inverted pendulum is a one degree of freedom platform with many certain applications and it also acts as a testbed for simulation and verification of control algorithms. The control of angular dynamics of an aerodynamically actuated dual rotor inverted pendulum using nonlinear robust geometric control method has been considered. The system parameters are assumed unknown and the technique of nonlinear adaptation using manifold immersion is performed for their estimation. Reference tracking is obtained. The complete system simulation is performed and controller is tuned using rapid prototyping. The theoretically proposed controller is validated experimentally. The discrete time realization of control algorithm is implemented on digital controller, which is interfaced in real time with Simulink. The potential of proposed algorithm relies upon the flexibility in the structure of control algorithm and promising transient behavior of closed loop system dynamics.*

1. Introduction

Inverted pendulum platform is among challenging control systems. This platform has been a benchmark to practice various control techniques. A lot of research work has been dedicated to this system owing to highly nonlinear and complex dynamic structure of the system. There are various configurations available for the inverted pendulum system. The configurations include belt and cart mechanism, in which a cart movable on a railing balance the inverted pendulum. A rotary version is also available. However, a somewhat newer concept is that of a rod with double rotor-propellers assemblies, with thrust in opposite directions. This system also has challenging dynamics and is considered in this paper.

There are various control techniques available in literature for inverted pendulum system. Observer based fuzzy linear matrix inequality regulator for stabilization and tracking control of an aero pendulum is considered in [1] by including Takagi-Sugeno fuzzy modeling. A fuzzy PID controller for propeller pendulum is presented in [2]. Stabilizing a driven inverted pendulum using DLQR Control technique is presented in [3] with theoretical and experimental validation. A new different technique to control a driven inverted pendulum with PID method has been proposed in [4]. Sliding mode control of suspended pendulum is considered in [5], which gives very good performance. Nonlinear model-based parameter estimation and stability analysis of an aero inverted pendulum subject to digital delayed control is presented in [6]. Design of quadratic dynamic matrix control for driven inverted pendulum system has been investigated in [7]. Another technique for stabilization of a propeller driven pendulum is presented in [8]. Designing intelligent adaptive controller for nonlinear inverted pendulum dynamical system is given in [9]. Propeller-Pendulum for Nonlinear UAVs Control is considered in [10]. Design controller for a class of nonlinear pendulum dynamical system is presented in [11]. Pendulum positioning system actuated by dual motorized propellers is presented in [12].

Most of these techniques consider the linear system model or a reduced order model of the system. Moreover, controllers do not have many tunable parameters to gain much control over system responses. Many controllers suffer degradation of response as the operation conditions change or the system parameters vary with time. We have applied a robust adaptive nonlinear control algorithm that relies on robustification of reduced order system controller against full order system dynamics [13]. The controller is also robust against unknow system parameters and has a lot of free tunable parameters to gain control over feedback dynamic response of the system output.

2. Hardware Overview

System hardware is shown in Figure 1. This hardware consists of a pivoted rod. Lower end of the rod has a counter weight attached to it. The other end of the rod carries BLDC motors driven by a digital controller and electronic speed controllers (ESC). A battery supplies power to ESCs. The hardware is interfaces with Matlab using a data acquisition card by National Instruments. The complete functionality will be explained in later sections.

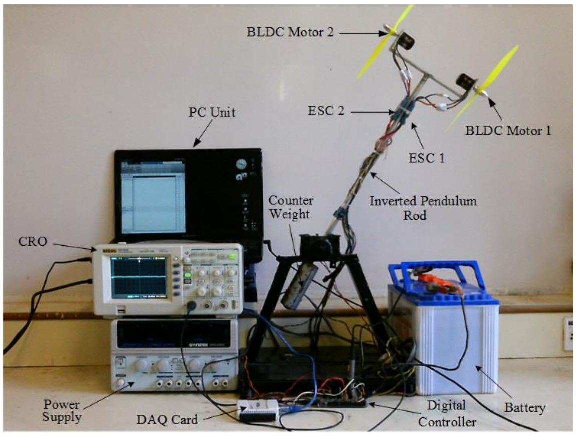

Figure 1. Aero pendulum hardware.

The two motor propellers produce two counter torques on the rod. The control problem as shown in Figure 2 is to design rod angle tracker, that monitors the angle of the rod and adjusts the speeds of the motors to keep this angle at a desired value with inadequate knowledge of system parameters.

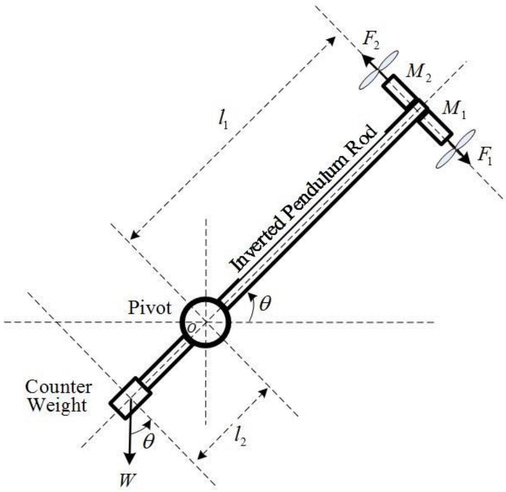

Figure 2. The control problem formulation

An inertial measurement unit (IMU) is used to measure the angle of the rod as shown in Figure 3.

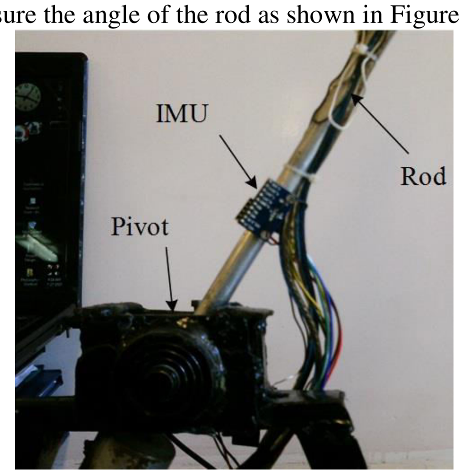

Figure 3. Rod angular displacement measurement.

3. Control Algorithm Synthesis

Consider a nonlinear parameter uncertain system,

$$\dot{\underline{p}} = \underline{s}\left(\underline{p}, u_e\right) = \underline{f}\left(\underline{p}\right) + \underline{g}\left(\underline{p}\right) u_e \tag{1}$$

where $\underline{p} \in \mathbb{R}^n$ and $u_e \in \mathbb{R}^m$. The state vector $\underline{p}$ evolves on a smooth manifold P of dimension n, which is spanned by tangential manifold to the system map $\underline{s}$. The system map $\underline{s}$ in (1) has been decomposed into a drift vector field $\underline{f}(.)$ and a controlled vector field $\underline{g}$. In (1), $u_e \in U\left(\underline{p}\right)$ is the system forcing function with U a state dependent input set which belongs to the control bundle $\bigcup_{\underline{p} \in \mathrm{P}} U\left(\underline{p}\right)$. The topological manifold immersion based nonlinear control approach involves defining a reduced order exosystem. The state trajectories of the exosystem evolve on a C^{∞} submanifold $\mathrm{Q} \subset \mathrm{P}$. The problem of controller design then boiled down to synthesize a control law that dynamically immerses the state trajectories of full order system to the manifold Q. Let us consider an exosystem with state vector $\underline{q} \in \mathbb{R}^q$ with $q < n$, which contains origin in its reachable set. This can be achieved by defining the vector field $\underline{\Upsilon}\left(\underline{q}\right)$ of the exosystem that governs the evolution of $\underline{q}$ as given by (2).

$$\dot{\underline{q}} = \underline{\Upsilon}\left(\underline{q}\right) \tag{2}$$

Defining a smooth submanifold for the exosystem of (2) as:

$$\mathrm{Q} = \left\{\underline{p} \in \mathbb{R}^n \middle| \underline{p} = \underline{\psi}\left(\underline{q}\right); \underline{q} \in \mathbb{R}^q\right\} \tag{3}$$

The controlled integral curves of system map $\underline{s}$ can be attracted by the submanifold Q if partial differential (4) along with the condition in (5) is satisfied [13].

$$\underline{f}\left(\underline{\psi}\left(\underline{q}\right)\right) + \underline{g}\left(\underline{\psi}\left(\underline{q}\right)\right)\wp\left(\underline{\psi}\right) = L_{\Upsilon}\underline{\psi} \tag{4}$$

$$\underline{q}(t) = \underline{0} \quad \forall \ \underline{q}(0) \in \mathbb{R}^2 \text{ as } t \to \infty \tag{5}$$

Here $L_{\underline{\Upsilon}}\underline{\psi} = (\nabla_{\underline{q}}\underline{\psi})\underline{\Upsilon}\left(\underline{q}\right)$ is the so-called Lie derivative. Also $\wp\left(\underline{\psi}\left(\underline{q}\right)\right) = v\left(\underline{\psi}\left(\underline{q}\right), 0\right)$ on the submanifold Q and $u = v\left(\underline{p}, \zeta\left(\underline{p}\right)\right)$ is the synthesized feedback control law that renders Q attractive. $\zeta(.)$ is the implicit description of Q and it is given by parameterized form in (6).

$$\zeta\left(\underline{p}\right) = \underline{p} - \underline{\psi}\left(\underline{q}\right) = 0 \tag{6}$$

Introducing state variable $\hbar$ to define "off" the submanifold Q dynamics given by:

$$\dot{\hbar} = L_{\underline{s}}\zeta\Big|_{u=\vartheta(\underline{p},\hbar)} = \left(\frac{\partial \zeta}{\partial \underline{q}}\right)\underline{s}\left(\underline{p},\vartheta\left(\underline{p},\hbar\right)\right) \quad (7)$$

In terms of $\hbar$ and any constant $\alpha > 0$, the synthesized controller ϑ the system mapping is given by,

$$\underline{\dot{p}} = \underline{s}\left(\underline{p},\vartheta\left(\underline{p},\hbar\right)\right) \quad (8)$$

For any general system of form,

$$\begin{aligned} \dot{p}_1 &= \underline{\xi}_1(p_1) + \underline{\xi}_2(\underline{p}_1)p_2 \\ \dot{p}_2 &= \underline{\varphi}(\underline{p})^T\underline{\lambda}_1 + \lambda_2 u \end{aligned} \quad (9)$$

where $\underline{\xi}_i(.)$ and $\underline{\varphi}(.)$ are smooth mappings, λ_i are unknown parameters and $\dot{p}_1 = \underline{\xi}_1(\underline{p}_1)$ is globally stable, then for constants $\varepsilon > 0$ and $k > 0$, the geometric adaptive estimates of λ_i are given by [9].

$$\dot{\hat{\underline{\lambda}}} = -\left(I + \nabla_{\hat{\lambda}}\underline{\upsilon}\right)^{-1}\begin{pmatrix} \left(\nabla_{\underline{p}}\underline{\upsilon}\right)\left(\underline{\xi}_1(\underline{p}_1) + \underline{\xi}_2(\underline{p}_1)p_2\right) \\ +\dfrac{\partial \underline{\upsilon}}{\partial p_2}\left(-kp_2 - \varepsilon L_{\underline{\xi}}V_1(\underline{p}_1)\right) \end{pmatrix} \quad (10)$$

and the corresponding geomantic synthesized control law is given by,

$$u = -\left(\hat{\lambda}_2 + \upsilon_2(\underline{p},\hat{\underline{\lambda}}_1)\right)\begin{pmatrix} kp_2 + \varepsilon L_{\underline{\xi}}V_1(\underline{p}_1) \\ +\varphi(\underline{p})^{\mathrm{T}}\left(\hat{\underline{\lambda}}_1 + \underline{\upsilon}_1(\underline{p})\right) \end{pmatrix} \quad (11)$$

The vector $\underline{\upsilon} = \left[\underline{\upsilon}_1(\underline{p}) \quad \upsilon_2(\underline{p},\hat{\underline{\lambda}}_1)\right]^{\mathrm{T}}$ is given by:

$$\underline{\upsilon}_1(\underline{p}) = \gamma_1\int_0^{p_2}\varphi(\underline{p}_1,\eta)d\eta \quad (12)$$

$$\begin{aligned} \upsilon_2(\underline{p},\hat{\underline{\lambda}}_1) &= \gamma_2\left(k\frac{p_2}{2} + \varepsilon L_{\underline{\xi}}V_1(\underline{p}_1)p_2\right) \\ &+ \gamma_2\int_0^{p_2}\varphi(\underline{p}_1,\eta)^{\mathrm{T}}\left(\hat{\underline{\lambda}}_1 + \underline{\upsilon}_1(\underline{p}_1,\eta)\right)d\eta \end{aligned} \quad (13)$$

Here $V_1(\underline{p}_1)$ is any mapping such that for some class-K function $\kappa(.)$, we have,

$$L_{\underline{\xi}}V_1(\underline{p}_1) \le -\kappa(\underline{p}_1) \quad (14)$$

and $\gamma_1 > 0$, $\gamma_2 > 0$ are constants [13].

4. System Dynamics and Control

Consider the experimental testbed of Aero Pendulum mechanism in the Figure 2. If θ denotes angle of the rod and ω denotes the angular speed the rod then the system state variables for angular dynamics are described by (15).

$$\underline{p} = \left[p_1 \quad p_2 \quad p_3\right]^{\mathrm{T}} = \left[\theta \quad \dot{\theta} \quad \omega\right]^{\mathrm{T}} \quad (15)$$

If we consider the second order curve fit for the static thrust calibration of motors against its driving signals, then the dynamics of system are described by following system,

$$\begin{aligned} \underline{f}\left(\underline{p}\right) &= \left[p_2 \quad -k_1p_2 + k_2p_3^2 \quad -p_3k_3\right]^{\mathrm{T}} \\ \underline{g}\left(\underline{p}\right) &= \left[0 \quad 0 \quad k_4\right]^{\mathrm{T}} \\ \underline{s}\left(\underline{p},u_e\right) &= \left[p_3 \quad -k_1p_2 + k_2p_3^2 \quad -p_3k_3 + k_4u_e\right]^{\mathrm{T}} \end{aligned} \quad (16)$$

Now (2) and (8) evaluate to following expressions.

$$\underline{\Upsilon}\left(\underline{q}\right) = \left[q_2 \quad -k_1 + k_2\varsigma_1^2\right]^{\mathrm{T}} \quad (17)$$

$$\underline{p} = \underline{\psi}\left(\underline{q}\right) = \left[q_1 \quad q_2 \quad \psi_1\left(q_1,q_2\right)\right]^{\mathrm{T}} \quad (18)$$

$$\vartheta\left(\underline{p},\hbar\right) = \frac{-\alpha\hbar + \dot{\varsigma}_1 + k_3p_3}{k_4} \quad (19)$$

The reduced order system is given by (20).

$$\begin{aligned} \dot{\hbar} &= -\alpha\hbar \\ \dot{p}_1 &= p_2 \\ \dot{p}_2 &= -k_1p_2 + k_2\varsigma_1^2 \\ \dot{p}_3 &= -\alpha\hbar + \dot{\varsigma}_1 \end{aligned} \quad (20)$$

The system in (20) immerses to system described by (21).

$$\begin{aligned} \dot{p}_1 &= p_2 \\ \dot{p}_2 &= -k_1 + k_2\varsigma_1^2 \end{aligned} \quad (21)$$

Let us consider feedback linearization of (21) as,

$$\varsigma_1 = \sqrt{u} : u > 0 \quad (22)$$

The immersion control law is given by,

$$\vartheta\left(\underline{p},\hbar\right) = \frac{-\alpha\hbar + \dot{\varsigma}_1 + k_3p_3}{k_4} \quad (23)$$

Using (21) and (22) we get,

$$\underline{\dot{p}} = \left[p_2 \quad -k_1p_2 + k_2u\right]^{\mathrm{T}} \quad (24)$$

For the estimation of unknown parameters in (24), using the results in (9) through (14), we get.

$$\begin{aligned} &\underline{\xi}_1(p_1) = 0, \quad \underline{\xi}_2(\underline{p}_1) = 1, \\ &\lambda_1 = k_1, \quad \lambda_2 = k_2 > 0, \quad \varphi(\underline{p}) = -1 \end{aligned} \quad (25)$$

$$L_{\underline{\xi}}V_1(\underline{p}_1) = 2\underline{p}_1 \quad (26)$$

$$\underline{\upsilon} = \begin{bmatrix} c_1p_2 \\ c_2p_2^2 +_1 c_3\hat{\lambda}_1p_2 + c_4p_1p_2 \end{bmatrix} \quad (27)$$

$$\nabla_{\hat{\lambda}}\underline{\upsilon} = \begin{bmatrix} 0 & 0 \\ -\gamma_2p_2 & 0 \end{bmatrix} \quad (28)$$

$$\nabla_{\underline{p}}\underline{\upsilon} = \frac{\partial \underline{\upsilon}}{\partial p_1} = \left[0 \quad 2\varepsilon\gamma_2p_2\right]^{\mathrm{T}} \quad (29)$$

$$\frac{\partial \underline{\upsilon}}{\partial p_2} = \left[-\gamma_1 \quad k\gamma_2p_2 + 2\gamma_2kp_1 - \gamma_2\hat{\lambda}_1 + \gamma_1\gamma_2p_2\right]^{\mathrm{T}} \quad (30)$$

The parameter estimates in (10) leads us to,

$$\dot{\underline{\hat{\lambda}}} = \begin{bmatrix} c_5 p_1 + c_6 p_2 \\ c_7 p_1^2 + c_8 p_2^2 + c_9 p_1 p_2 + c_{10} p_2 \hat{\lambda}_1 + c_{11} p_1 \hat{\lambda}_1 \end{bmatrix} \qquad (31)$$

The control law in terms of estimates parameters is given by,

$$u = -\left(\hat{\lambda}_2 + \upsilon_2(\underline{p}, \hat{\lambda}_1)\right)\left(kp_2 + 2\varepsilon p_1 - \hat{\lambda}_1 - \upsilon_1(\underline{p})\right) \qquad (32)$$

At the last the reference tracking is achieved by modifications of control law as,

$$\vartheta'(\underline{p}, \hbar) = \frac{-\alpha\hbar + \dot{\varsigma}_1 + k_3 p_3}{k_4} + \sigma(t) \qquad (33)$$

A typical classical proportional derivative tracker law can be used to follow reference command as given by,

$$\sigma(t) = \Xi(e(t)) \qquad (34)$$

$$\Xi(.) = -\frac{k_3}{k_4}\left(k_p(.) + k_d \frac{d(.)}{dt}\right) \qquad (35)$$

The control law in (33) can be split to produce drive signals of two motors according to the splitter algorithm given by (36).

$$\vartheta_1 = \begin{cases} \left|\vartheta' - \vartheta'_m\right| + \vartheta_{1_{th}} & if \quad \left(\vartheta' - \vartheta'_m\right) > 0 \\ \vartheta_{1_{th}} & if \quad \left(\vartheta' - \vartheta'_m\right) < 0 \end{cases}$$
$$\vartheta_2 = \begin{cases} \left|\vartheta' - \vartheta'_m\right| + \vartheta_{2_{th}} & if \quad \left(\vartheta' - \vartheta'_m\right) < 0 \\ \vartheta_{2_{th}} & if \quad \left(\vartheta' - \vartheta'_m\right) > 0 \end{cases} \qquad (36)$$

$\vartheta_{i_{th}}$ is the upper threshold value of i^{th} motor drive signal dead zone and ϑ'_m is the mean value of signal ϑ'.

5. Simulation and Testing

The values of various system parameters are given in Table 1. Using these values, the closed loop system is simulated in Figure 4.

Table 1. System parameter values.

Parameter	Value	Parameter	Value
k_3	145	c_4	0.018
k_4	7.15	c_5	-0.25
k	61	c_6	-214
γ_1	4.1	c_7	5×10^{-4}
γ_2	1.0	c_8	845.0
ε	2.1×10^{-3}	c_9	0.25
c_1	-6.25	c_{10}	-750.0
c_2	72.0	c_{11}	-0.05
c_3	-3.25	c_{12}	0.125

The simulation result for yaw reference tracking response is shown in Figure 5. The response is stable with zero steady state error.

The simulation result for the yaw rate response is shown in Figure 6. The simulation result for manipulated variables response is shown in Figure 7. The yaw rate decays to zero within 1.5 seconds in Figure 6. The magnitude of manipulated variables is within practical limits in Figure 7 and drives the plant output to the desired reference signal.

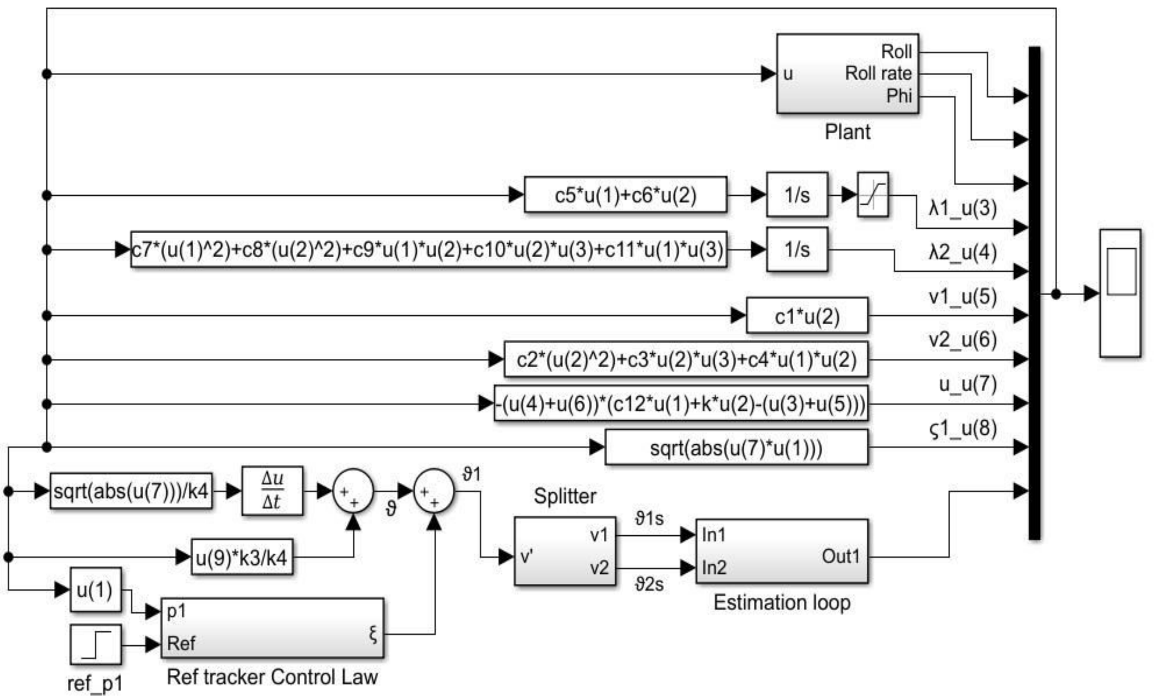

Figure 4. Simulation of closed loop system.

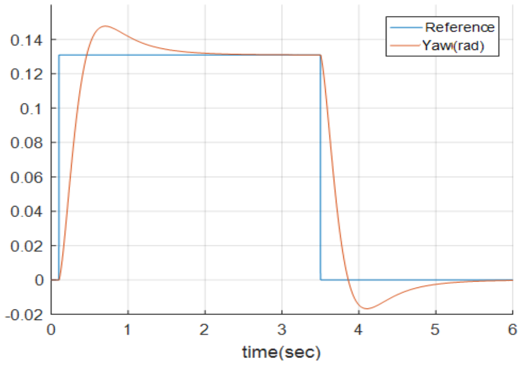

Figure 5. Yaw reference tracking

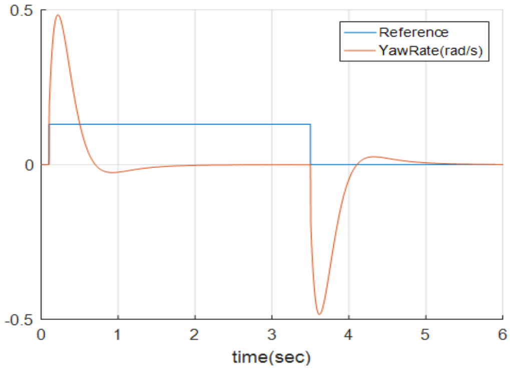

Figure 6. Yaw rate response

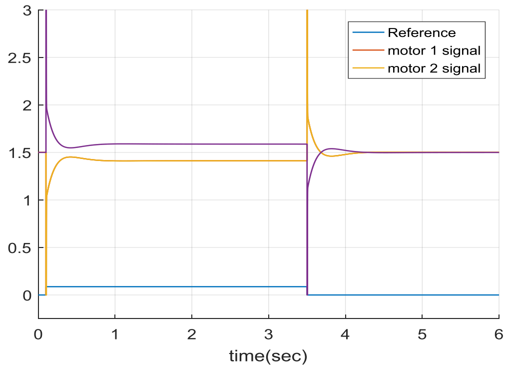

Figure 7. Manipulated variables' response

The experimental RCP Simulink model of the closed loop system with reference tracker is shown in Figure 8. The schematic representation of experimental hardware setup is shown in Figure 12.

The experimental result for yaw reference tracking response, yaw rate and manipulated variables are shown in Figure 9, Figure 10 and Figure 11 respectively. Yaw reference response is stable with zero steady state error. Yaw rate response decays to zero within 1.5 seconds. The manipulated variables, have magnitude within practical limits and drives the plant output to the desired reference signal.

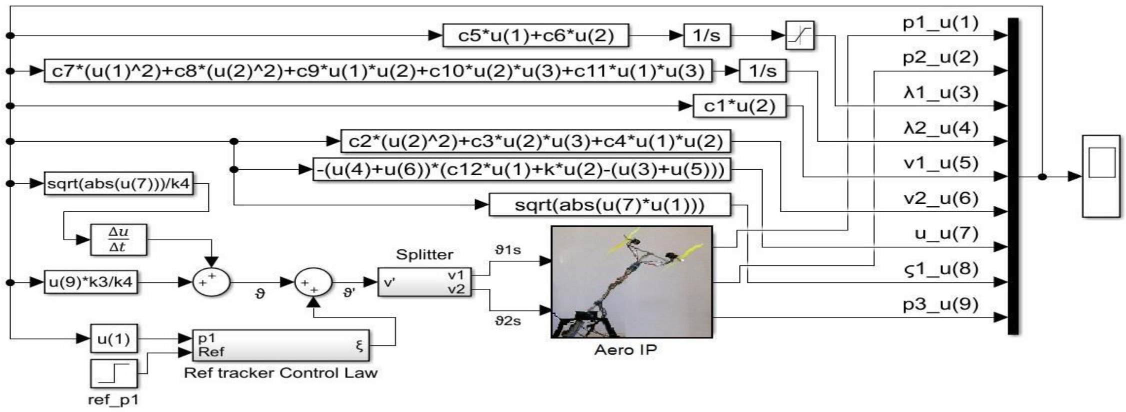

Figure 8. RCP mode of operation of Hardware

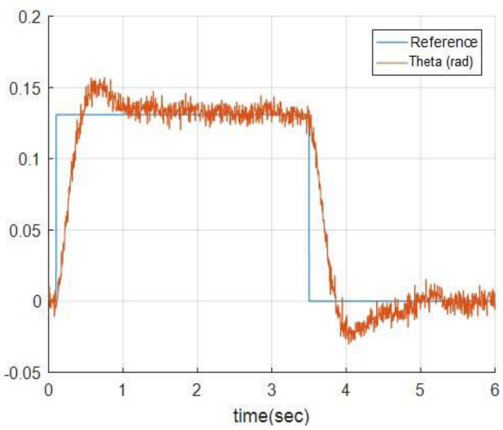

Figure 9. Yaw reference tracking

6. Conclusions

A robust nonlinear geometric controller for the dynamics of an aero pendulum has been presented. System parameters are considered unknown and they are estimated using nonlinear adaptation. The proposed control technique is simulating in Simulink. The theoretical technique is tested in real time using digital controllers interfaced in real time with Simulink. The control algorithm has a lot of free tunable parameters. The results showed promising behavior of closed loop system in the presence of parameters uncertainties. Moreover, a greater control of closed loop system dynamics is possible owing to the flexibility in control algorithm.

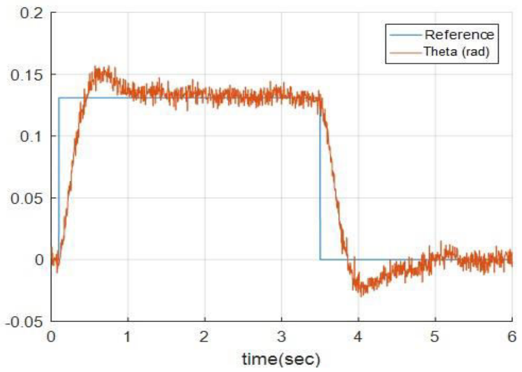

Figure 10. Yaw rate response

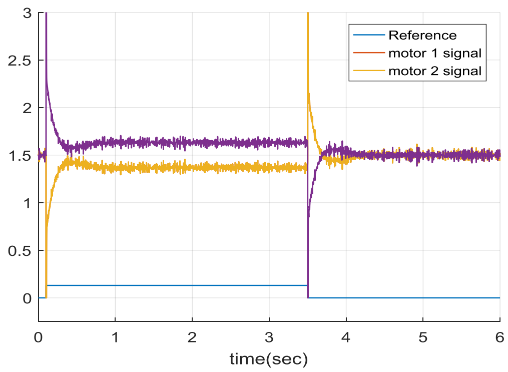

Figure 11. Manipulated variables' response

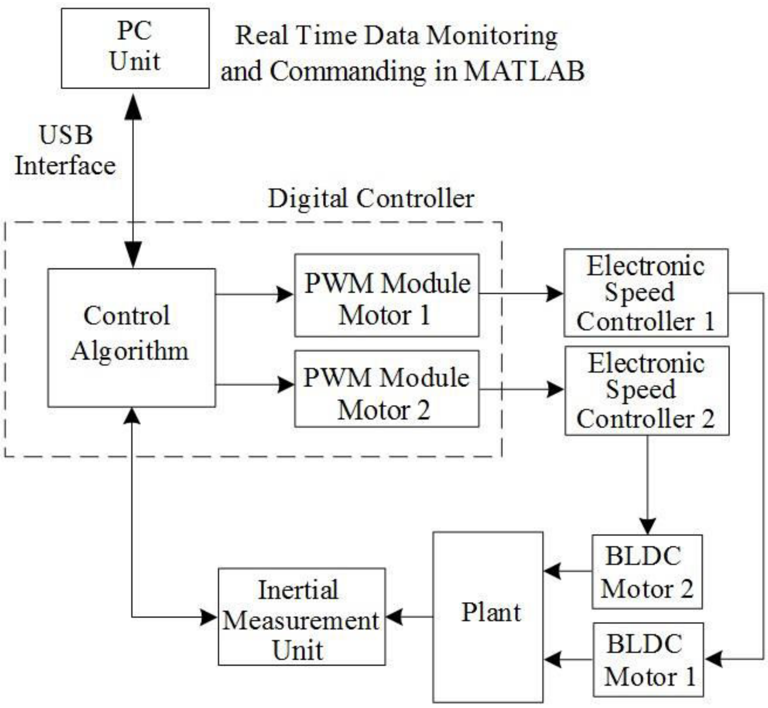

Figure 12. Schematic representation of hardware.

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