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A study on ζαω-closed sets using grill space

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Abstract : *In this paper we apply the notion of ζαω-closed sets and obtain a new class of generalized closed sets using grills. Also we investigate the properties of ζαω-closed sets.*

Keywords: *Grill, topology τ_G, operator Φ, ζω-open, ζαω–closed.*

1 Introduction

The concepts of grill topological spaces depended on the two operators are Φ and Ψ. This concept was first introduced by Choquet [3] in 1947. In [1], Roy and Mukherjee defined and studied a typical topology associated rather naturally to the existing topology and a grill on a given topological space. N. Chandramathi[7] have defined new class of generalized closed sets using grills. Introduced and investigated the notions of ζω-open sets, ζ-semi-open sets in grill topological spaces.

2 Preliminaries

Throughout this paper (X, τ) (or simply X) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

Definition 2.1. A subset A of (X, τ) is called
(i) semi-generalized closed (briefly sg-closed) set[8] if scl(A) ⊆ U whenever A ⊆ U and U is semi open set in (X, τ).
(ii) generalized semi-closed (briefly gs-closed) set [9]if scl(A) ⊆ U whenever A ⊆ U and U is open set in (X, τ).
(iii) generalized α-closed (briefly gα-closed) set[4] if αcl(A) ⊆ U whenever A ⊆U and U is α-open set in (X, τ)

Definition 2.2. [3] A collection ζ of non empty subsets of a spaces X is called grill on X if
(i) A ∈ ζ and A ⊆ B ⊆ X ⇒ B ∈ ζ
(ii) A, B ⊆ X and A ∪ B ∈ ζ ⇒ A ∈ ζ or B ∈ ζ.

Definition 2.3. [1] Let (X, τ) be a topological space and ζ be a grill on X. We define a mapping Φ : P (X) → P (X) denoted by Φ_ζ (A, τ) (for A ∈ P (X)) or Φ_ζ (A) or simply Φ_ζ (A), called the operator associated with the grill ζ and the topology τ, and is defined by $\Phi(A) = \varphi_\zeta (X, \tau, \zeta) = \{x \in X : A \cap U_x \in \zeta, \forall U_x \in \tau(x)$ for each $A \in P (X)\}$

Definition 2.4. [1]Let ζ be grill on a space X. We define a map Ψ : P (X) →P (X) by Ψ(A) = A ∪ Φ(A) for all A ∈ P (X).

Definition 2.5. [1]Corresponding to a grill ζ on a topological space (X, τ) there exists a unique topology τ_ζ (say) on X given by $\tau_\zeta = \{U \subseteq X : \Psi(X \setminus U) = (X \setminus U)\}$ where for any all A ⊆ X, $\Psi(A) = A \cup \Phi(A) = \tau_\zeta - cl(A)$.

Definition 2.6. [2] Let X be a space and (φ 6=)A ⊆ X. Then A = {B ⊆ X : A ∩ B ≠ φ} is a grill on X called principal grill generated by A.

Definition 2.7. [7] Let (X, τ) be a topological space and ζ be any grill on X. Then a subset A of X is called ζω-closed if ψ(A) ⊆ U whenever A ⊆ U and U is ζ-semi open in X. A subset A of X is called ζω-open if X \ A is ζω-closed.

3 ζαω-closed sets

Definition 3.1. Let (X, τ) be a topological space and ζ be any grill on X. Then a subset A of X is called ζαω-closed if ψ(A) ⊆ U whenever A ⊆ U and U is ζ- ω-open in X. A subset A of X is called ζαω-open if X \ A is ζαω-closed.

Theorem 3.2. Every closed set in (X, τ) is ζαω-closed in (X, τ, ζ).

Proof. Let A be any closed set and U be any ζ-ω-open set such that cl(A) =A ⊆ U. Since A is closed. But ψ(A) ⊆ cl(A), we have ψ(A) ⊆ U whenever A ⊆ U. Hence A is ζαω-closed.
The converse of the above theorem is not true as seen from the following example.

Example 3.3. Let X = {a, b, c}, with τ = {φ, {a},X} and ζ = {{a}, {b}, {a, b}, {a, c}, {b, c},X}. Let A = {b} and U = {b, c} where U is ζω-open in X. Therefore φ(A) = {φ} and ψ(A) = A ∪ φ(A) = {b} ⊆ U. Then the set {b} is ζαω-closed but not closed.

Theorem 3.4. Every αω-closed set in (X, τ) is ζαω-closed in (X, τ, ζ).

Proof. Let A be any αω closed set and U be any ζω-open set containing A. Since every ζω-open set is ω-open and A is αω-closed we have, cl(A) ⊆ U. But ψ(A) ⊆ cl(A). Thus we have, ψ(A) ⊆ U whenever A ⊆ U. Hence A is ζαω-closed.

The converse of the above theorem is not true as seen from the following example.

Example 3.5. Let X = {a, b, c} with τ = {φ, {b}, {c}, {b, c}, {a, b}, X} and ζ = {a}, {b}, {a, b}, {a, c}, {b, c}, X}. Let A = {b} and U = {b, c} where U is ζω-open in X. Therefore φ(A) = {b} and ψ(A) = A ∪ φ(A) = {b} ⊆ U. Then the set {b} is ζαω-closed set but not αω-closed set in (X, τ).

Remark 3.6. In a grill space (X, τ, ζ) ζαω-closed sets are generalized of αω-closed set which itself is a generalized closed set in (X, τ, ζ).

Theorem 3.7. Every ζαω-closed set in (X, τ, ζ) is ζg-closed in (X, τ, ζ).

Proof. Let A ⊆ U, U is open and hence it is ζω-open. Since A is ζαω-closed, we have ψ(A) ⊆ U. But φ(A) ⊆ ψ(A) ⊆ U. Hence A is ζg-closed.

Example 3.8. Let X = {a, b, c} with τ = {φ, {a}, X} and ζ = {{a}, {b}, {a, b}, {a, c}, {b, c}, X}. Then the set {a, b} is ζg-closed but not ζαω-closed

Remark 3.9. In the case [X] principal grill generated by X, it is know that τ = τ[X] so that any [X]- ζαω-closed set becomes an αω-closed set and vice-versa.

Theorem 3.10. Every ζαω-closed set in (X, τ, ζ) is a gα-closed set in (X, τ).

Proof. Let A be a ζαωis -closed set in (X, τ, ζ). Then ψ(A) ⊆ U whenever A ⊆ U and U is ζω-open. We know that cl(A) ⊆ U. Then every closed set is α-closed set. We have αcl(A) ⊆ U and also every α-open set is ω-open. αcl(A) ⊆ U whenever A ⊆ U and U is α-open.

The converse of the above theorem is not true as seen from the following example.

Example 3.11. Let X = {a, b, c} with {φ, {a}, X} and ζ = {X, {a}, {c}, {a, c}}. Then the set {b} is gα-closed set in (X, τ) but not ζαω-closed in (X, τ, ζ).

Theorem 3.12. Every ζαω-closed set in (X, τ, ζ) is a gs-closed set in (X, τ).

Proof. Let A be a ζαω -closed set in (X, τ, ζ). Then ψ(A) ⊆ U whenever A ⊆ U and U is ζω-open. We know that cl(A) ⊆ U. Then every closed set is semi-closed set. We have scl(A) ⊆ U and also every open set is ω-open. There-fore scl(A) ⊆ U whenever A ⊆ U and U is open.

The converse of the above theorem is not true as seen from the following example.

Example 3.13. Let X = {a, b, c} with τ = {φ, {b}, {c}, {b, c}, {a, c}, X} and ζ = {X, {a}, {c}, {a, c}, {a, b}, {b, c}}. Then the set {a, c} is gs-closed set in (X, τ) but not ζαω-closed set in (X, τ, ζ).

Theorem 3.14. Every ζαω-closed set in (X, τ, ζ) is a sg-closed set in (X, τ).

Proof. Let A be a ζαω-closed set in (X, τ, ζ). Then ψ(A) ⊆ U whenever A ⊆ U and U is ζω-open. We know that cl(A) ⊆ U. Then every closed set is semi-closed set. We have scl(A) ⊆ U and also every semi-open set is ζω-open. Therefore scl(A) ⊆ U whenever A ⊆ U and U is semi-open.

The converse of the above theorem is not true as seen from the following example.

Example 3.15. Let X = {a, b, c} with τ = {φ, {a}, {c}, {a, c}, {a, b}, X} and ζ = {{a}, {b}, {a, b}, {a, c }, {b, c}, X}. Then the set {a, b} is sg-closed set in (X, τ) but not ζαω-closed set in (X, τ, ζ).

Theorem 3.16. Let (X, τ, ζ) be a topological space and ζ be a grill on X. Then for a subset A of X, the following are equivalent.

(i) A is ζαω-closed.
(ii) ψ(A) ⊆ U for ζω-open set U containing A.
(iii) For each x ∈ ψ(A), ζ({x} ∩ A) ≠ φ.
(iv) ψ(A) \ A contains no nonempty ζω-closed set of (X, τ, ζ).
(v) φ(A) \ A contains no nonempty ζω-closed set of (X, τ, ζ).

Proof. (i) ⇒ (ii) Let A be a ζαω-closed. Then clearly, ψ(A) ⊆ U whenever A ⊆ U and U is ζω-open in X.

(ii) ⇒ (iii) suppose x ∈ ψ(A). If ζωcl({x}∩ A) = φ, then A ⊆ X \ ζωcl({x}) is a ζω-open set. By assumption ψ(A)\A ⊆ X \ζωcl({x}), which is a contradiction to x ∈ ψ(A). Hence ζωcl({x}) ∩ A ≠ φ, This prove (iv).

(iii) ⇒ (iv) Assume that F ⊆ φ(A). where F is ζω-closed and F ≠ φ. This gives F ⊆ φ(A). This contradicts (iv).

(iv) ⇒ (v) It follows from the fact that ψ(A) \ A = φ(A) \ A.

(v) ⇒ (i) Let A ⊆ U where U is ζω-open such that φ(A) * U. This gives φ(A) ∩ (X − U) = φ or φ(A) \ [X \ (X \ U)] = φ. This gives φ(A) \ A ≠φ. Moreover, φ(A)\A = φ(A)∩(X \U) is ζω-closed set in x. Since φ(A) = cl(φ(A)) is closed in X and X \ U ∈ ζωc(X). Also, φ(A) \ U ⊆ φ(A) \ A. This gives that φ(A) \ A contains a non empty ζω-closed set. This contradicts (v). This completes the proof. I

corollary 3.17. Let (X, τ) be a T_1 space and ζ be a grill on X. Then every ζαω-closed set is τ_ζ -closed.

corollary 3.18. Let (X, τ, ζ) be a grill topological space and A be a ζαω-closed set. The following are equivalent.
(i) A is τ_ζ -closed.

(ii) ψ(A) \ A is ζω-closed set in (X, τ, ζ).
(iii) φ(A) \ A is ζω-closed set in (X, τ, ζ).
Proof. Let A be a τ_ζ -closed. Then φ(A) \ A = ψ(A) \ A gives ψ(A) \ A = φ. This proves that ψ(A) \ A is ζω-closed.

(ii) ⇒ (iii) Since φ(A) \ A = ψ(A) \ A and so φ(A) \ A is ζω-closed in X. (iii) ⇒ (i) Let φ(A)\A be a ζω-closed set. Now A is ζαω-closed and by theorem 3.16 (v). φ(A)\A contains no nonempty ζω-closed set. Therefore, φ(A)\A = φ. This proves φ(A) = A and hence A is τ_ζ -closed.

Theorem 3.19. In a grill space (X, τ, ζ) an ζαω-closed set and τ_ζ -dense set in itself is αω-closed.

Proof. Suppose A is τ_ζ -dense in itself and ζαω-closed in X. Let U be any ζω open set containing A, then ψ(A) ⊆ U. Since A is τ_ζ -dense in itself then, φ(A) = cl(φ(A)) = ψ(A) = αcl(A), we get αcl(A) ⊆ U whenever A ⊆ U. This proves that A is αω-closed.

corollary 3.20. If (X, τ, ζ) is any grill space where ζ = P (X) \ {φ} then A is ζαω-closed if and only if A is αω-closed.

Proof. The proof follows from the fact that ζ = P (X)\{φ}, φ(A) = αcl(A) ⊃ A and so every subset of X is τ_ζ -dense set in itself.
The following theorem gives another characterization of ζαω-closed set.

Theorem 3.21. Let (X, τ, ζ) be a grill space. Then A ⊆ X is ζαω-closed if and only if A = F \ N, where F is τ_ζ -closed and N contains no nonempty ζω-closed set

Proof. If A is ζαω-closed set then by Theorem 3.16, N = ψ(A) \ A contains no nonempty ζω-closed set. Let F = ψ(A), then F is τ_ζ -closed set and F − N = A ∪(φ(A)) \ (φ(A) \ A) = A.

Conversely, Let U be any ζω-open set in X containing A, then F \ N ⊆ U implies F ∩ (X \ U) ⊆ F ∩ (X \ (F \ N)) = F ∩ (X \ F) ∪ N = F ∩ N ⊆ N. By hypothesis A ⊆ F and φ(F) ⊆ F as F is τ_ζ -closed gives φ(A) ∩ (X \ U) ⊆ φ(F) ∩ (X \ U) ⊆ F ∩ (X \ U) ⊆ N where φ(A) ∩ (X \ U) is ζω-closed set. By hypothesis φ(A) ∩ (X \ U) = φ or ψ(A) ⊆ U implies that A is ζαω-closed set.

Theorem 3.22. Let (X, τ, ζ) be a grill space. Then every subset of X is ζαω-closed if and only if every ζω-open set is τ_ζ -closed.

Proof. Suppose that every subset of X is ζαω-closed. Let U be a ζω-open set then U is ζαω-closed and ψ(A) ⊆ U. Hence U is τ_ζ -closed.

Conversely, Suppose that every ζω-open set is τ_ζ -closed. Let A be nonempty subset of X contained in a ζω-open set U. Then ψ(A) ⊆ ψ(U) implies ψ(A) ⊆ U. This Proves that A is ζαω-closed.

Theorem 3.23. A set A is ζαω-open if and only if F ⊆ τ_ζ -int(A) whenever F is ζω-closed and F ⊆ A.

Proof. Suppose that F ⊆ τ_ζ -int(A), where F is ζω-closed and F ⊆ A. Let A^c ⊆ U, where U is ζω-open. Then U^c ⊆ A and U^c is ζω-closed. Therefore, U^c ⊆ τ_ζ -int(A). Since U^c ⊆ τ_ζ -int(A), we have (τ_ζ -int(A))^c ⊆ U. That is ψ(A^c) ⊆ U, since ψ(A^c) = (τ_ζ -(int(A))^c. Thus A^c is ζω-closed, that is A is ζαω-open.
Conversely, Suppose that A is ζαω-open. F ⊆ A and F is ζω-closed. Then F^c is ζω-open and A^c ⊆ F^c. Therefore ψ(A_c) ⊆ F^c and so F ⊆ τ_ζ -int(A), since ψ(A^c) = (τ_ζ -int(A))^c.

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