μ-β-generalized α-open sets in generalized topological spaces

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Abstract: *In this paper, we have introduced a new class of sets in generalized topological spaces called μ-β-generalized α-open sets. Also we have investigated some of their basic properties.*

Keywords:*Generalized topology, generalized topological spaces, μ-α-open sets, μ-β-generalized α-open sets.*

1. Introduction

The concept of generalized topological spaces is introduced by A. Csaszar [1] in 2002. He also introduced many μ-open sets like μ-semi open sets, μ-pre-open sets, μ-β-open sets, μ-α-open sets, μ-regular open sets etc., in generalized topological spaces. In this paper, we have introduced a new class of sets in generalized topological spaces called μ-β-generalized α-open sets. Also we have investigated some of their basic properties and produced many interesting theorems.

2. Preliminaries

Definition 2.1:[1] Let X be a nonempty set. A collection μ of subsets of X is a generalized topology (or briefly GT) on X if it satisfies the following:

i. $\emptyset, X \in \mu$ and
ii. If $\{M_i : i \in I\} \subseteq \mu$, then $\cup_{i\in I} M_i \in \mu$

If μ is a GT on X, then (X, μ) is called a generalized topological space (or briefly GTS), and the elements of μ are called μ-open sets and their complement are called μ-closed sets.

Definition 2.2:[1] Let (X, μ) be a GTS and let $A \subseteq X$. Then μ-closure of A, denoted by $c_\mu(A)$, is the intersection of all μ-closed sets containing A.

Definition 2.3:[1] Let (X, μ) be a GTS and let $A \subseteq X$. Then μ-interior of A, denoted by $i_\mu(A)$, is the union of all μ-open sets contained in A.

Definition 2.4: [1] Let (X, μ) be a GTS. A subset A of X is said to be

i. μ-semi-open if $A \subseteq c_\mu(i_\mu(A))$
ii. μ-pre-open if $A \subseteq i_\mu(c_\mu(A))$
iii. μ-α-open if $A \subseteq i_\mu(c_\mu(i_\mu(A)))$
iv. μ-β-open if $A \subseteq c_\mu(i_\mu(c_\mu(A)))$
v. μ-regular open if $A = i_\mu(c_\mu(A))$

Definition 2.5: [3] Let (X, μ) be a GTS. A subset A of X is said to be

i. μ-regular generalized open if $i_\mu(A) \supseteq U$ whenever $A \supseteq U$, where U is μ-regular closed in X,
ii. μ-generalized open set if $i_\mu(A) \supseteq U$ whenever $A \supseteq U$, and U is μ-closed in X,
iii. μ-generalized α-open set if $\alpha i_\mu(A) \supseteq U$ whenever $A \supseteq U$, and U is μ-closed in X.

3. μ-β-generalized α-closed sets in generalized topological spaces

In this section we have introduced μ-β-generalized α-open sets in generalized topological spaces and studied some of their basic properties.

Definition 3.1: The complement A^c of a μ-β generalized α-closed set (briefly μ-βGαCS) in (X, μ) is called the μ-β generalized α-open set (briefly μ-βGαOS) in (X, μ).

Example 3.2: Let X= {a, b, c} and let μ = {Ø, {a}, {b}, {a, b}, X}. Then (X, μ) is a GTS. Now μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Let A = {a, b} then A^c = {c} is a μ-βGαCS. Hence A is a μ- βGαOS in (X, μ).

Theorem 3.3: Every μ-open set in (X, μ) is a μ-βGαOS in (X, μ) but not conversely.

Proof: Let A be a μ-open set in (X, μ). Then its complement A^c is a μ-closed set in X. Therefore A^c

is μ-βGαCS [4] in X and hence by Definition 3.1, A is a μ- βGαOS in (X, μ).

Example 3.4: Let X= {a, b, c, d} and let μ = {Ø, {a}, {c}, {a, c}, X}. Then (X, μ) is a GTS. Now μ-βO(X) = {Ø, {a}, {c}, {a, b}, {a, c}, {a, d}, {b, c}, {c, d}, {a, b, c}, {b, c, d}, {a, c, d}, {a, b, d}, X}. Then A = {a, b, c} and A^c = {d}. Hence A = {a, b, c} is a μ-βGαOS in (X, μ) as its complement is μ-βGαCS in X, but not a μ-open set as $i_\mu(A) = i_\mu(\{a, b, c\}) = \{a, c\} \neq A$.

Theorem 3.5: Every μ-α-open set in (X, μ) is a μ- βGαOS in (X, μ).

Proof: Let A be a μ-α-open in (X, μ). Then A^c is a μ-α-closed set in (X, μ). Then $c_\mu(i_\mu(c_\mu(A^c))) \subseteq A^c$. Now let $A^c \subseteq U$ where U is μ-β-open in (X, μ). Then $\alpha c_\mu(A^c) = A^c \cup c_\mu(i_\mu(c_\mu(A^c))) = A^c \cup A^c = A^c \subseteq U$, by hypothesis. Therefore $\alpha c_\mu(A^c) \subseteq U$. This implies A^c is μ-βGαCS in X. Hence A is a μ-βGαOS in (X, μ).

Theorem 3.6: Every μ-regular open set in (X, μ) is a μ-βGαOS in (X, μ) but not conversely.

Proof: Let A be a μ-regular open set in (X, μ). As every μ-regular open set is μ-open, A is μ-open in (X, μ). Therefore by Theorem 3.3 A is a μ- βGαOS in (X, μ).

Example 3.7: Let X= {a, b, c} and let μ= {Ø, {a}, {b}, {a, b}, X}. Then (X, μ) is a GTS. Now μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Then A = {a, b} and A^c ={c} and A = {a, b} is a μ-βGαOS in (X, μ) as A^c is a μ- βGαCS in X, but not a μ-regular open set as $i_\mu(c_\mu(A)) = i_\mu(c_\mu(\{a, b\})) = X \neq A$.

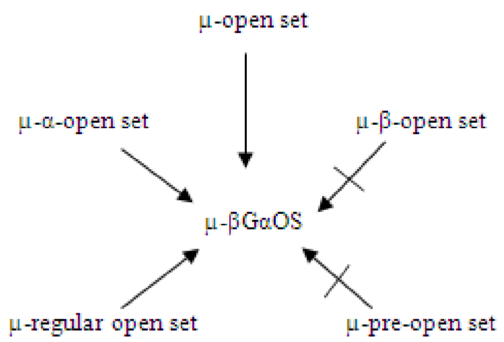
Remark 3.8: A μ-pre-open set is not a μ-βGαOS in (X, μ) in general.

Example 3.9: Let X= {a, b, c} and μ= {Ø, {a, b}, X}. Then (X, μ) is a GTS. Now μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Let A = {b, c}. Then A is a μ-pre-open in (X, μ) as $i_\mu(c_\mu(A)) = i_\mu(c_\mu(\{b, c\})) = X$ and $A \subseteq X$, but not a μ- βGαOS in (X, μ).

Remark 3.10: A μ-β-open set is not a μ-βGαOS in (X, μ) in general.

Example 3.11: Let X= {a, b, c} and μ= {Ø, {a, b}, X}. Then (X, μ) is a GTS. Now μ-βO(X) = {Ø, {a}, {b}, {a, b}, {b, c}, {a, c}, X}. Let A = {b, c}. Then A is a μ-β-open set in (X, μ) as $c_\mu(i_\mu(c_\mu(A))) = c_\mu(i_\mu(c_\mu(\{b, c\}))) = X$ and $A \subseteq X$, but not a μ-βGαOS in (X, μ).

In the following diagram, we have provided relations between various types of open sets.



Theorem 3.12: Let A and B be μ-βGαOSs in (X, μ) such that $i_\mu(A) = \alpha i_\mu(A)$ and $i_\mu(B) = \alpha i_\mu(B)$, then $A \cap B$ is a μ-βGαOS in (X, μ).

Proof: Let $A \cap B \supseteq U$, where U is a μ-β-closed set in X. Then $A \supseteq U$ and $B \supseteq U$. since A and B are μ-βGαOSs, $\alpha i_\mu(A) \supseteq U$ and $\alpha i_\mu(B) \supseteq U$. Now, $\alpha i_\mu(A \cap B) \supseteq i_\mu(A \cap B) = i_\mu(A) \cap i_\mu(B) = \alpha i_\mu(A) \cap \alpha i_\mu(B) \supseteq U \cap U = U$. Hence, $A \cap B$ is a μ-βGαOS in (X, μ).

Theorem 3.13: If $\alpha i_\mu(A) \subseteq B \subseteq A$ and A is a μ-βGαOS, then B is also a μ-βGαOS in (X, μ).

Proof: Let $\alpha i_\mu(A) \subseteq B \subseteq A$ and A is a μ-βGαOS in X. Then $(\alpha i_\mu(A))^c \supseteq B^c \supseteq A^c$ and so $A^c \subseteq B^c \subseteq \alpha c_\mu(A^c)$ as $\alpha c_\mu(A^c) = (\alpha i_\mu(A))^c$. If A is a μ-βGαOS then A^c is a μ-βGαCS. By Theorem 3.13 in [4] we have B^c is a μ-βGαCS and hence B is a μ-βGαOS in (X, μ).

Theorem 3.14: A subset A of X is a μ-βGαOS iff $F \subseteq \alpha i_\mu(A)$ we have $F \subseteq A$ and F is a μ-β closed set in (X, μ).

Proof: **Necessity:** Let A be a μ-βGαOS. Then A^c is a μ-βGαCS. Let F be a μ-β closed set and $F \subseteq A$. Then F^c is a μ-β-open set and $A^c \subseteq F^c$. Therefore $\alpha c_\mu(A^c) \subseteq F^c$, by hypothesis. This implies that $(\alpha i_\mu(A))^c \subseteq F^c$. That is $F \subseteq \alpha i_\mu(A)$.

Sufficiency: Let $A^c \subseteq U$ where U is a μ-β-open set in (X, μ). Then U^c is a μ-β-closed set and $U^c \subseteq (A^c)^c = A$. Let $U^c = F$. Hence by hypothesis, $F \subseteq \alpha i_\mu(A)$. This implies that $(\alpha i_\mu(A))^c \subseteq F^c$. That is $\alpha c_\mu(A^c) \subseteq F^c = U$. Therefore A^c is a μ-βGαCS and thus A is a μ-βGαOS in (X, μ).

Theorem 3.15: If A is both a μ-β-closed set and a μ-βGαOS in (X, μ), then A is a μ-α-open set in (X, μ).

Proof: Let A be both a μ-β-closed set and a μ-βGαOS in (X, μ). Then, $\alpha i_{\mu}(A) \supseteq A$ as $A \supseteq A$. we have $A \supseteq \alpha i_{\mu}(A)$. Therefore, $A = \alpha i_{\mu}(A)$. Hence A is a μ-α-open set in (X, μ).

Theorem 3.16: Every subset of X is a μ-βGαOS in X iff every μ-β-closed set in X is μ-α-open in X.

Proof: **Necessity:** Let A be a μ-β-closed set in X and by hypothesis, A is a μ-βGαOS in X. Hence by Theorem 3.15, A is a μ-α-open set in X.

Sufficiency: Let A be a subset of X and U be a μ-β-closed set such that $A \supseteq U$, then by hypothesis, U is μ-α-open. This implies that $\alpha i_{\mu}(U) = U$ and $\alpha i_{\mu}(A) \supseteq \alpha i_{\mu}(U) = U$. Hence $\alpha i_{\mu}(A) \supseteq U$. Thus A is a μ-βGαOS in (X, μ).

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4. References

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