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Minimum Spanning Steiner Fuzzy Tree Algorithm

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Abstract: *In this paper, we present an algorithm to find a minimum spanning tree of fuzzy graph G, and also for calculating the value of minimum spanning Steiner fuzzy tree of G, when costs c_{ij} are associated with its edges. The algorithm uses random sampling in combination with Steiner points for verifying a minimum spanning Steiner fuzzy tree problem.*

Keywords: *Shortest Spanning Steiner fuzzy tree (SSSFT), Steiner points, rectilinear problem.*

1. INTRODUCTION

The Shortest Spanning Steiner fuzzy tree (SSSFT) of a fuzzy graph has various applications in cases where roads (gas pipelines, electric power lines, etc) are to be used to connect "n" points together in such a way so as minimize the total length of road that has to be constructed. If the "n" points to be connected are on a Euclidean plane, they can be represented as vertices of a complete fuzzy graph G with arc costs being the straight line distances between the corresponding end points. The Shortest Spanning Steiner fuzzy tree(SSSFT) of G is then (provided no road junction outside the given "n" points are allowed) the required minimum –cost road network. If junctions outside the given "n" points are allowed, then an even shorter road network may be possible and finding it is a problem known as Fuzzy Steiner's Problem.

While a spanning fuzzy tree spans all vertices of a given fuzzy graph, a *Steiner fuzzy tree* spans a given subset of vertices. In the Steiner minimal fuzzy tree problem, the vertices are divided into two parts: terminals and non-terminal vertices. The terminals are the given vertices which must be included in the solution. The cost of a Steiner fuzzy tree is defined as the total edge weight. A Steiner fuzzy tree may contain some non-terminal vertices to reduce the cost. Let *V* be a set of vertices. In general, we are given a set $L \subset V$ of terminals and a metric defining the distance between any two vertices in *V*. The objective is to find a connected fuzzy sub-graph spanning all the terminals of minimal total cost. Since the distances are all nonnegative in a metric, the solution is a fuzzy tree structure.

2.The Steiner Problem

The Steiner fuzzy network problem is the optimization problem of finding a fuzzy tree $F_T = (X', E')$ in G such that $X \subset X'$ and the weights $W(e)$ is as small as possible, given a set X of vertices in an undirected graph G = (X ,*E*) with a non-negative weight function *W* defined on *E.* The fuzzy tree F_T is a Steiner fuzzy tree of the set X. The vertices in $(X' - X)$ are the Steiner points of X with respect to *Ts.*

Definition : 1

A Steiner fuzzy tree with respect to a pair of vertices in an undirected fuzzy network is any shortest fuzzy path *P* between them, and the Steiner points of this pair with respect to *P* are the intermediate vertices of this fuzzy path.

Example : 1

Considering the 4 points shown in Fig. 1(a), the shortest spanning fuzzy tree is as shown, whereas the introduction of two new points S_1 and S_2 in the middle, produces a shortest spanning fuzzy tree spanning all 6 points, whose total length is less than that of the previous shortest spanning fuzzy tree shown in Fig.1(b).

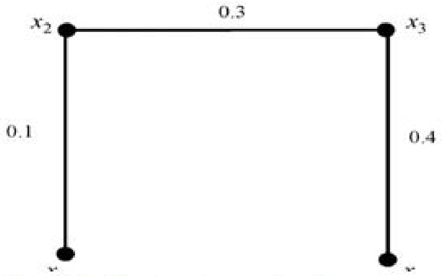

Fig. 1(a). Shortest spanning fuzzy tree

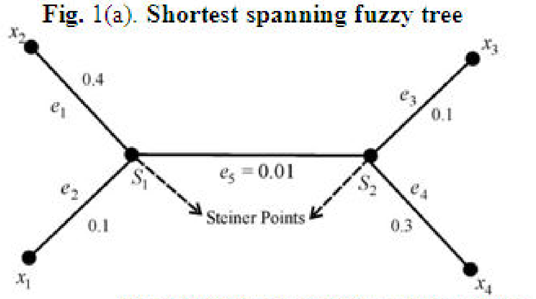

Fig.1 (b) Shortest Steiner Fuzzy Tree

Thus, in the original Steiner problem, as many Steiner points as necessary could be added anywhere in the fuzzy graph, in order to produce the shortest fuzzy tree spanning the specified set of *n* points. This resulting spanning fuzzy tree is then called a shortest Steiner fuzzy tree.

2.1 PROPERTIES OF THE SHORTEST STEINER FUZZY TREE

- For a Steiner point S_i, the degree d $(S_i) = 3$. It can be easily shown by geometrical considerations that the angle between links incident at any Steiner point must be 120°, and the angle links incident at any Steiner point S_i . This point is, therefore, the "centre" (Steiner centre) of an imaginary triangle whose vertices are the other 3 points, to which S_i, is linked in the shortest Steiner fuzzy tree. Some of the points forming the vertices of this triangle may themselves be other Steiner points. In Fig 1(b), Steiner points S_2 is the Steiner centre of the imaginary triangle with vertices x_3, x_4 and S_1.

For a vertex $x_i \in X$, $d(x_i) \leq 3$. If $d(x_i) = 3$, then the angle between any two of the three links incident at x_i must be 120°, and if $d(x_i) = 2$ the angle between the two links must be greater than or equal to 120°.

- The number K of Steiner points in a shortest Steiner fuzzy tree is
  $0 \leq K \leq n-2$, where $n = |X|$..

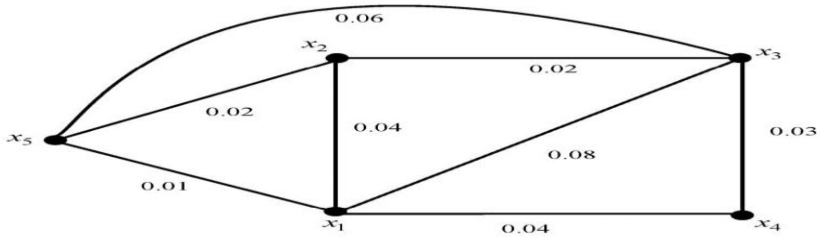

Fig.2 Fuzzy Graph

If X' is a proper subset of V, a minimum spanning fuzzy tree of the fuzzy sub-graph induced by the set X' need not be a Steiner fuzzy tree with respect to V. The weight of a minimum spanning fuzzy tree of the fuzzy sub-graph induced by the set X' = {x_1, x_2, x_3 , x_4 } is seen to be 0.09. But it is possible to obtain a fuzzy tree F_T that spans these four vertices and vertex x_5 such that the weight of the edge set of F_T is only 0.08.

Theorem :1.1

Let {1,2, ... ,n} be the vertex set of the complete fuzzy graph G with a non-negative weight function W satisfying the triangle property
$W(\{i,j\}) \leq W(\{i,k\})+W(\{k,j\})$ for all vertices i,j and k. If X is a set of m vertices in fuzzy network G, there exists a Steiner fuzzy tree for X in the fuzzy network that contains atmost $(n - 2)$ Steiner points.

Proof :

Let the number of Steiner points of X with respect to a fuzzy tree F_T be q. Then F_T has $(m + q)$ vertices and $(m + q - 1)$ edges. If x is the average degree (in F_T) of a Steiner point and y is the average degree (in F_T) of a non-Steiner point, the degree sum $qx + my = 2(m + q - 1)$

By triangle property, for $x \geq 3$, and y is atleast one. Hence $q \leq m - 2$.

2. Algorithm for minimum spanning fuzzy tree enumeration

Step 1: The input is a set Q of m vertices in a connected fuzzy network G = (X ,E) of order n in which a nonnegative weight is defined on each edge.

Step 2: Construct the complete fuzzy network G' = (X ,E') in which the weight of an edge joining two vertices is the shortest distance between them.

Step 3: Let $f = \{S \subset (X-Q): |S| \leq (m-2)\}$. For each subset S in f, find an minimum spanning fuzzy tree of the fuzzy sub graph of G' induced by $Q \cup S$ on G'. Among the fuzzy trees thus obtained, select a fuzzy tree F_t of minimum weight.

Step 4: Construct a fuzzy tree F_T from F_t , by replacing each edge joining two vertices by the set of edges in a shortest fuzzy between them. Those vertices of F_T 'that are not in Q will form a set of Steiner points for Q.

Example:2.1

Find the Steiner trees with respect to $V_1 = \{x_3, x_6, x_7\}$ in the fuzzy network.

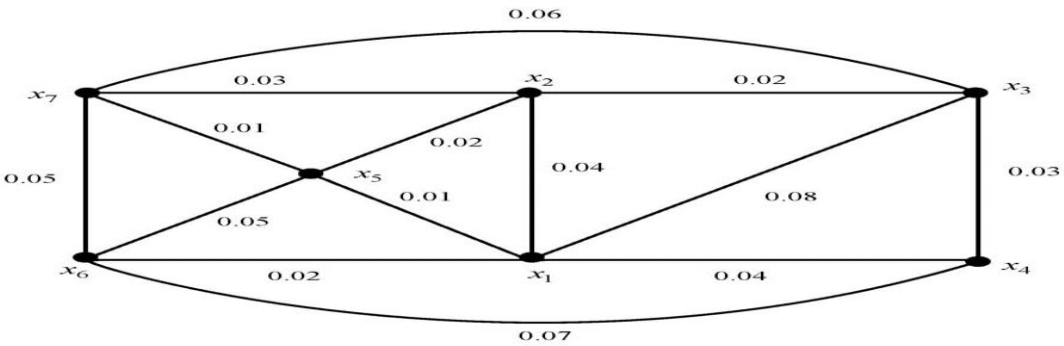

Fig.3 Fuzzy Network

The shortest distance matrix for this fuzzy network $[D(F_N)]$ is Table-1

Table 1

$D(F_N) =$

0	0.03	0.05	0.04	0.01	0.02	0.02
0.03	0	0.02	0.05	0.02	0.05	0.03
0.05	0.02	0	0.03	0.04	0.07	0.05
0.04	0.05	0.03	0	0.05	0.06	0.06
0.01	0.02	0.04	0.05	0	0.03	0.01
0.02	0.05	0.07	0.06	0.03	0	0.04
0.02	0.03	0.05	0.06	0.01	0.04	0

Since $m = 3$, S is any subset of $\{x_1, x_2, x_4, x_5\}$ with atmost one element. There are five choices for $(Q \cup S)$;

$Q_1 = \{x_3, x_6, x_7\}$
$Q_2 = \{x_3, x_6, x_7, x_1\}$
$Q_3 = \{x_3, x_6, x_7, x_2\}$
$Q_4 = \{x_3, x_6, x_7, x_4\}$
$Q_5 = \{x_3, x_6, x_7, x_5\}$

The minimum weight spanning fuzzy trees of the fuzzy sub graphs of G' induced by these sets have weights 0.09, 0.09, 0.09, 0.12 and 0.08 respectively. So we take the fuzzy sub graph induced by V_5 on the complete fuzzy network G'. **In** this fuzzy sub-graph, a minimum spanning fuzzy tree $F_{t'}$ as shown in Fig.4(a).

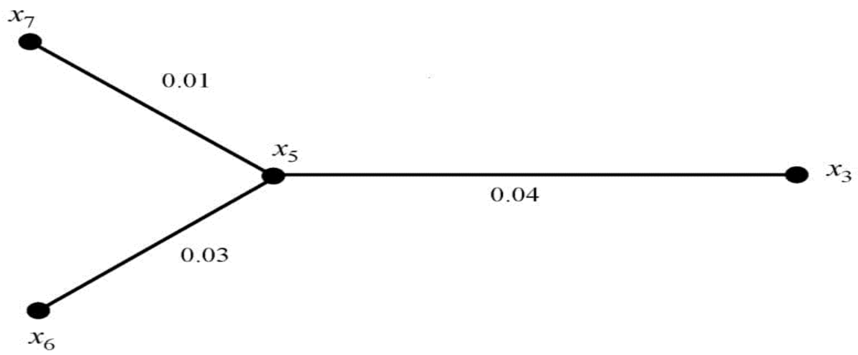

Fig. 4(a) Minimum spanning fuzzy tree $F_{T'}$

The weight of the edge in $F_{t'}$ between x_3 and x_5 is 0.04. This shortest fuzzy path is x_5 — x_2 — x_3. Likewise the edge between x_6 and x_5 is replaced by the fuzzy path x_5 — x_1 — x_6. Thus the minimum Steiner fuzzy tree with respect to $\{x_3, x_6, x_7\}$ is as shown in _Fig.4(b). The Steiner points are x_1, x_2

and x_5.

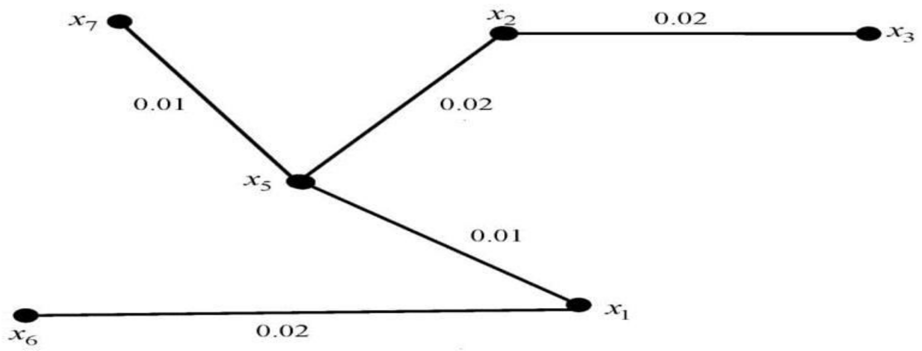

Fig. 4(b) Steiner fuzzy tree F_{T_S}

Example :2.2 (6-point rectilinear problem)

Let a fuzzy graph G be found so that the set X of its vertices is the set of distinct grid line intersections and the links of G correspond to grid lines joining two intersection points. Fig. *5(b)* shows an example of a shortest Steiner fuzzy tree of a 6-point rectilinear problem and, for comparison, fig. 5(a) shows the shortest spanning fuzzy tree of this problem.

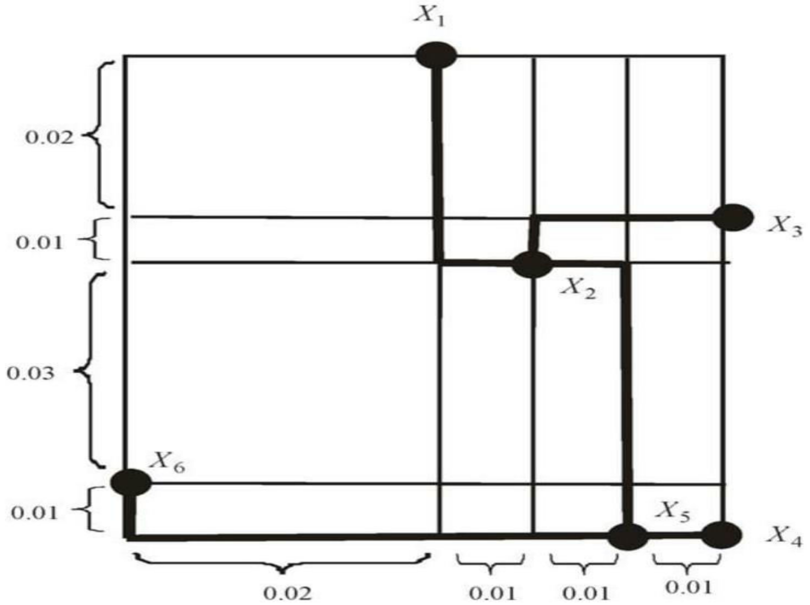

Fig. 5(a). Shortest spanning fuzzy tree

Length = 0.18

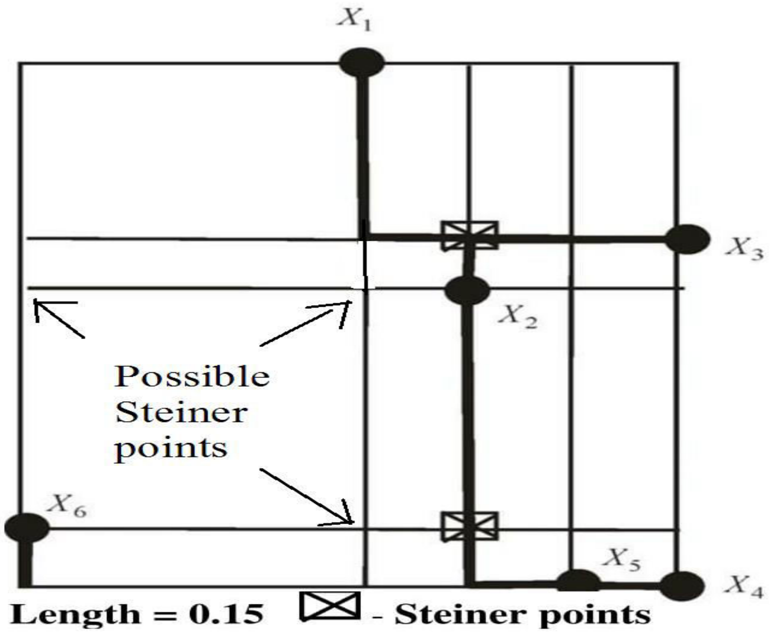

Fig 5(b) Shortest Spanning Steiner Fuzzy Tree

Conclusion

The minimum weight spanning fuzzy trees of the fuzzy sub graphs of G' induced by the set $\{x_3, x_6, x_7, x_5\}$ having weight 0.08 . So we take the fuzzy sub graph induced by V_5 on the complete fuzzy network. Thus the minimum Steiner fuzzy tree with respect to $\{x_3, x_6, x_7\}$ is as shown in Fig.4(b). The Steiner points are x_1, x_2 and x_5. Also , the value of the shortest spanning fuzzy tree of 6-point rectilinear problem is 0.18 and the value of the shortest spanning Steiner fuzzy tree of 6-point rectilinear problem is 0.15. Hence we conclude that the distance or cost of transportation problem or optimization problem are minimized by Steiner points, and are useful to reduce the time or money. However, these algorithms are computationally inefficient procedures (for more than 10 vertices) even though they are much better than the complete enumeration of shortest spanning fuzzy tree's of all fuzzy sub-graphs G' of G.

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