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Dd-Distance in Graphs

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***Abstract**: For two vertices u and v of a graph G, the usual distance d(u, v), is the length of the shortest path between u and v. In this paper we introduced the concept of Dd-distance by considering the degrees of various vertices presented in the path, in addition to the length of the path We study some properties with this new distance. We define the eccentricities of vertices, radius and diameter of G with respect to the Dd-distance. First we prove that the new distance is a metric on the set of vertices of G. We compare the usual, detour and Dd-distances of two vertices u, v of V.*

***Keywords:** Detour distance, Dd-distance, Dd-Eccentricity, Dd-Radius and Dd-Diameter.*

1. Introduction

By a graph G, we mean a non-trivial finite undirected connected graph without multiple edges and loops. Following standard notations (for any unexplained notation and terminology we refer [2]) $V(G)$ or V is the vertex set of G and $E(G)$ or E is the edge set of $G = G(V, E)$. Let u, v be two vertices of G. The *standard* or *usual distance* $d(u, v)$ between u *and* v is the length of the shortest $u - v$ path in G. Chartrand el al [3] introduced the concept of *detour distance* in graphs as follows: For two vertices u, v in a graph G, the *detour distance* $D(u, v)$ is defined as the length of the longest $u - v$ path in G.

In this article we introduce a new distance, which we call as *Dd-distance* between any two vertices of a graph G, and study some of its properties. This distance is significantly different from other distances. In some of the earlier distances, only path length was considered. Here we, in addition, consider the degree of u and v vertices present in a $u - v$ path while defining its length. Using this length we define the *Dd*-distance. Chartand et al introduced the concept of *detour distance* by considering the length of the longest path between u and v. Kathiresan et al introduced the concept of *superior distance* and *signal distance*. In some of these distances only the lengths of various paths were considered.

2. *Dd*-distance

Definition 2.1

If u, v are vertices of a connected graph G, *Dd*-length of a u-v path is defined as
$D^{Dd}(u, v) = D(u, v) + \deg(u) + \deg(v)$.

Example 2.2

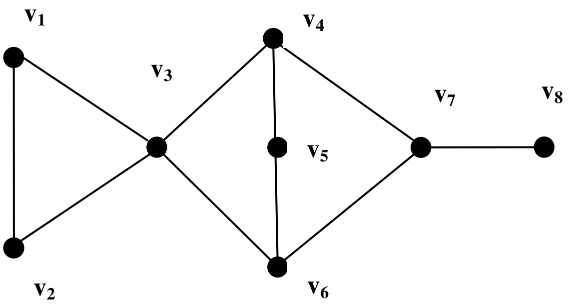

Figure 2.1 A graph G

In figure 2.1, if $u = v_1$, $v = v_8$, then $D^{Dd}(u, v) = D(u, v) + \deg(u) + \deg(v) = 7 + 2 + 1 = 10$.

Theorem 2.3

If G is any connected graph, then the D^{Dd}-distance is a metric on the set of vertices of G.

Proof:

Let G be a connected graph and $u, v \in V(G)$. Then it is clear by definition, that $D^{Dd}(u, v) \geq 0$ and $D^{Dd}(u, v) = 0$ which implies $u = v$. Also we have $D^{Dd}(u, v) = D^{Dd}(v, u)$. Thus it remains to prove that D^{Dd} satisfies the triangle inequality.

Case(i): Let $u, v, w \in V(G)$. Let P and Q be $u - w$ and $w - v$ detour paths in G respectively such that $D^{Dd}(u, w) = l_{Dd}(P)$ and $D^{Dd}(w, v) = l_{Dd}(Q)$. Let $R = P \cup Q$ be the $u - v$ detour path obtained by joining P&Q at w of length greater than one. Then,
$D^{Dd}(u, w) + D^{Dd}(w, v) = [l(P) + \deg(u) + \deg(w)] + [l(Q) + \deg(w) + \deg(v)]$
$= l(P \cup Q) + \deg(u) + \deg(v) + 2\deg(w) = l(P \cup Q) + \sum_{u,v=x\in P \cup Q} \deg(x) + 2\deg(w) > D^{Dd}(u, v)$.

Therefore $D^{Dd}(u, w) + D^{Dd}(w, v) > D^{Dd}(u, v)$.

Case(ii): Suppose $R = P \cup Q$ be the $u - v$ detour path of length one. Then we take a vertex other than u & v such that either $u = w$ or $v = w$.

Subcase(i): Suppose $u = w$. Then $D^{Dd}(u, w) = 0$.
Therefore $D^{Dd}(u, w) + D^{Dd}(w, v) = 0 + l(Q) + \deg(w) + \deg(v) = l(P \cup Q) + \deg(u) + \deg(v)$.
$D^{Dd}(u, w) + D^{Dd}(w, v) = D^{Dd}(u, v)$.

Subcase(ii): Suppose $v = w$. Similar to subcase(i).
Thus the triangular inequalities hold and hence D^{Dd} is a metric on the vertex set. ■

Corollary 2.4

For any three vertices u, v, w of a graph G, $D^{Dd}(u, v) \leq D^{Dd}(u, w) + D^{Dd}(w, v) - \deg(w)$

Proof:

Let G be a connected graph.
$D^{Dd}(u, w) + D^{Dd}(w, v) = l(P \cup Q) + \sum_{u,v=x \in P \cup Q} \deg(x) + 2\deg(w) \geq D^{Dd}(u, v) + 2\deg(w)$.
$D^{Dd}(u, w) + D^{Dd}(w, v) \geq D^{Dd}(u, v) + \deg(w)$
Therefore $D^{Dd}(u, v) \leq D^{Dd}(u, w) + D^{Dd}(w, v) - \deg(w)$. ■

Proposition 2.5

In a tree T, two distinct vertices u, v are adjacent if and only if $D^{Dd}(u, v) = \deg(u) + \deg(v) + 1$

Proof:

Let u, v are adjacent. This implies that $D(u, v) = 1$. By definition, $D^{Dd}(u, v) = D(u, v) + \deg(u) + \deg(v) = \deg(u) + \deg(v) + 1$. Conversely, If $D^{Dd}(u, v) = \deg(u) + \deg(v) + 1$. This implies that $D(u, v) = 1$. Hence u, v are adjacent. ■

Proposition 2.6

If G is a connected graph with two distinct vertices u, v are adjacent then $D^{Dd}(u, v) \leq 3(n - 1)$

Proof:

Let u, v are adjacent. This implies that $D(u, v) \leq n - 1$. By definition, $D^{Dd}(u, v) = D(u, v) + \deg(u) + \deg(v) \leq \deg(u) + \deg(v) + n - 1$. Since $D(u, v) \leq n - 1$, $\deg(v) \leq n - 1$ for any vertices u and v. Hence $D^{Dd}(u, v) \leq 3(n - 1)$. ■

3. *Dd*-Eccentricity, *Dd*-Radius and *Dd*-Diameter

We begin with definitions for *Dd*-Eccentricity, *Dd*-Radius and *Dd*-Diameter.

Definition 3.1

The *Dd-eccentricity* of any vertex v, $e^{Dd}(v)$, is defined as the maximum distance from v to any other vertex, i.e., $e^{Dd}(v) = \max\{D^{Dd}(u, v) : u \in V(G)\}$

Definition 3.2

Any vertex u for which $D^{Dd}(u, v) = e^{Dd}(v)$ is called *Dd-eccentric* vertex of v .
Further, a vertex u is said to be *Dd-eccentric vertex* of G if it is the *Dd*-eccentric vertex of some vertex.

Definition 3.3

The *Dd-radius,* denoted by $r^{Dd}(G)$, is the minimum *Dd-eccentricity* among all vertices of G i.e., $r^{Dd}(G) = \min\{e^{Dd}(v) : v \in V(G)\}$. Similarly the *Dd-diameter,* $D^{Dd}(G)$, is the maximum *Dd-eccentricity* among all vertices of G.

Definition 3.4

The *Dd-center* of G, $C^{Dd}(G)$, is the subgraph induced by the set of all vertices of minimum *Dd-eccentricity.* A graph is called *Dd-self-centered* if $C^{Dd}(G) = G$ or equivalently $r^{Dd}(G) = D^{Dd}(G)$.Similarly, the set of all vertices of maximum *Dd*-eccentricity is the *periphery* of G.

Remark 3.5

For a tree T, the *Dd*-center need not be connected.

Remark 3.6

For any non-trivial connected graph G, $r^{Dd}(G) \leq D^{Dd}(G) \leq 2\, r^{Dd}(G)$.

Proof:

By definition of radius and diameter, $r^{Dd}(G) \leq D^{Dd}(G)$ and we have to claim other inequality. Let u and v be two vertices with $D^{Dd}(u, v) = D^{Dd}(G)$. Let $w \in C^{Dd}(G)$. Hence $e^{Dd}(w) = r^{Dd}(G)$ and so $D^{Dd}(w, x) \leq r^{Dd}(G)$ for any other vertex x of G. Now $D^{Dd}(G) = D^{Dd}(u, v) \leq D^{Dd}(u, w) + D^{Dd}(w, v) \leq r^{Dd}(G) + r^{Dd}(G) = 2\, r^{Dd}(G)$. Hence $D^{Dd}(G) \leq r^{Dd}(G)$. It completes the proof. ■

Theorem 3.7

If u, v are two adjacent vertices of a connected graph G, with $e^{Dd}(u) \geq e^{Dd}(v)$ then $e^{Dd}(u) - e^{Dd}(v) \leq 2(n - 1)$.

Proof:

Let w be an eccentric vertex of u such that $D^{Dd}(u, w) = e^{Dd}(u)$. Then,
$e^{Dd}(u) \leq D^{Dd}(u, v) + D^{Dd}(v, w) - \deg(v)$ by corollary:2.4
$\leq D^{Dd}(u, v) + e^{Dd}(v) - \deg(v)$ since $e^{Dd}(v) \geq D^{Dd}(v, w)$.
Since u and v are adjacent, $D^{Dd}(u, v) \leq \deg(u) + \deg(v) + n - 1$
$e^{Dd}(u) - e^{Dd}(v) \leq D^{Dd}(u, v) - \deg(v)$
$= \deg(u) + n - 1$
$e^{Dd}(u) - e^{Dd}(v) \leq 2(n - 1)$. ■

Theorem 3.8

For every tree $u, v \in T$, $D^{Dd}(u, v) > D(u, v)$.

Proof:

In a tree $D^{Dd}(u, v) = D(u, v)$
We know that $D^{Dd}(u, v) = D(u, v) + \deg(u) + \deg(v)$
This implies that $D^{Dd}(u, v) > D(u, v)$.

The following results are immediate for a cycle and complete graph. ■

Theorem 3.9

(i) For a cycle C_n, $D^{Dd}(u, v) = n+3 \ \forall \ u, v \in C_n$.
(ii) For a complete graph C_n, $D^{Dd}(u, v) = 3(n-1) \ \forall \ u, v \in K_n$.

Theorem 3.10

For any connected graph G, $D^{Dd}(u, v) = 3 \ \forall \ u, v \in G$, then G becomes a path of length 1 with two vertices.

Proof:

We have $D^{Dd}(u, v) = D(u, v) + \deg(u) + \deg(v)$. Since $|D^{Dd}(u,v)| = 3$, $D(u, v) = 3 - \{\deg(u) + \deg(v)\}$. Therefore $\deg(u) \leq 1$ and $\deg(v) \leq 1$. Now $\deg(u) + \deg(v) = 2$. Therefore $D(u, v) = 1$.This implies that u and v are the two vertices of length 1. ■

Theorem 3.11

For any connected graph G, if u and v are end vertices of G and $D(u, v) = n - 1$ then $D^{Dd}(u, v) = n - 1$.

Proof:

This result follows from the theorem 3.10. ■

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