

# Prediction of Load at Which Non Linearity Initiates in a Cantilever Beam

Vaisakh S<sup>1</sup> & Ruby Maria Syriac<sup>2</sup>

<sup>1</sup>PG Student, Mechanical Engineering Department, Mar Baselios College of Engineering and Technology, Trivandrum, Kerala, India.

<sup>2</sup>Assistant Professor, Mechanical Engineering Department, Mar Baselios College of Engineering and Technology, Trivandrum, Kerala, India.

---

**Abstract:** *Non linearity is a frequent occurrence in engineering structures and its study is being more widely accepted over linear experimental modal analysis. Nonlinear systems exhibit extremely complex behaviour's which the linear systems do not produce. Non Linearity can be induced by varying the geometry of the model and excitation like frequency, displacement. The cantilever system used for the experiment have two beams which are connected to each other. Hence in this system the resonance condition is the cause of the nonlinearity in the system. Both the beam interacts together to reach the resonance condition. At this condition the maximum deflection is obtained and lead to nonlinearity.*

**Index Terms**— *Nonlinear dynamics, Nonlinear normal modes, Modal analysis, Resonance.*

## 1. Introduction

Nonlinear systems exhibit extremely complex behaviour's which the linear systems do not produce. These phenomena include jumps, resonance captures, bifurcations, saturation, limit cycles, sub harmonic, super harmonic and internal resonances, modal interactions and chaos. Non-linear normal modes of vibration for a cantilever beam are assessed considering both the continuous framework and finite element models. Non Linearity can be induced by varying the geometry of the model. Another way of introducing nonlinearity could be increasing the amplitude of input excitation. The thin beam used is the cause of nonlinearity in the system. The load level at which the nonlinear behaviour initiates is obtained by using static analysis in Ansys.

W. Lacarbonara [1] constructed a one-dimensional consistent framework with cubic geometric non-linearity and quadratic geometric non-linearity. Carlos E.N. Mazzilli [2] assessed Non-linear normal modes of vibration for a hinged-hinged beam with fixed ends considering both the continuous framework and finite element models. Xinye Lia examined [3] the non-linear normal modes (NNMs) and their bifurcation of a complex two DOF

framework. The coupling and ground springs have both quadratic and cubic non-linearity all the while. [4] L. Renson modelled a full-scale rocket structure utilizing the finite element technique to identify the normal nonlinear modes. G. Kerschen [5], calculated the nonlinear normal modes and their oscillating frequencies can now be accomplished for unpredictable, real world aviation structures, like airframe of the Morane-Saulnier Paris flying machine. Nikolay V. Perepelkin [6] proposed a new technique for the forced resonance vibrations development in mechanical frameworks with internal resonance. The generalized theory of non-linear normal vibration modes by Shaw and Pierre, the modified Rauscher technique and the harmonic balance method are joined with another iterative computation methodology. Paolo Casini identified [7] few engineering frameworks present piecewise-linear characters, among them, the damaged beams with breathing splits are exceptionally compelling. The dynamics of such frameworks displays bifurcations at inner resonances portrayed by the onset of superabundant nonlinear normal modes with their individual modal shapes. Robert J. Kuethe [8] technique utilizes the Jacobian of the shooting function in a Newton-Raphson algorithm to locate the underlying conditions and integration period that outcome in a periodic response of the traditionalist equations of motion. E.H. Moussi [9] held an investigation of the Nonlinear Normal Modes (NNMs) of a two degrees of freedom mechanical framework with a bilateral elastic stop. Elvidio Gavassoni [10] created a hypothesis of nonlinear normal modes to explore the nonlinear vibrations of a discrete two-degree-of-freedom conceptual model of an offshore compliant explained tower. Ansys code development.

## 2. Ansys code development

### 2.1. Geometric Modelling

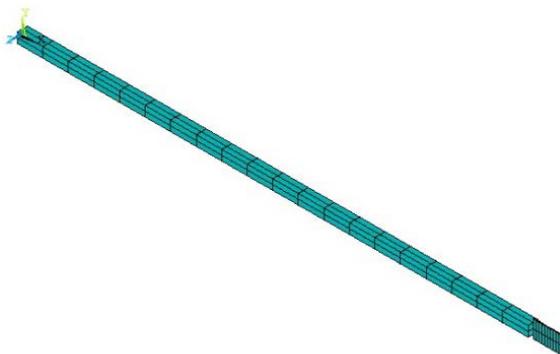
A cantilever beam used in the experiment can be considered as a model and we can apply the same principle for the other cantilever structures having

the geometric nonlinearity such as aircraft wings, long bridges, helicopter blades, etc. The detailed dimensions of the part considered for the analysis is given in Table 2.1.

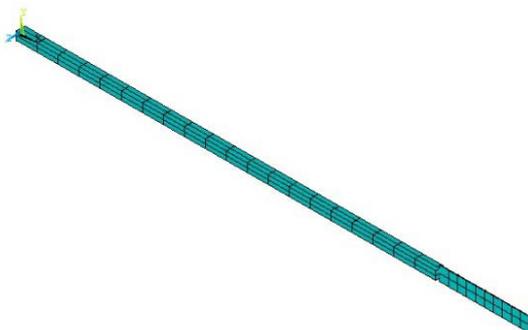
**Table 2.2 : Dimensions of the beams**

	Length(m)	Width(m)	Thickness(m)
Main beam	0.7	0.014	0.014
Thin beam 1	0.04	0.024	0.001
Thin beam 2	0.15	0.024	0.001

The cantilever beam is having two parts, the main beam or the thick beam and the thin beam section. These two beams are joined together and the structure is modelled for 4cm thin beam as shown in fig 2.1 and 15cm thin beam as shown in fig 2.2.



**Fig. 2.1 : Cantilever Beam of 4cm modelled for Analysis**



**Fig. 2.2 : Cantilever Beam of 15cm modelled for Analysis**

## 2.2. Material Model

The material used for the analysis is Stainless Steel for the thin beam and mild steel for the thin beam. Since forced simulation takes place in a variable load condition, a very detailed force dependent, nonlinear material model is required for the simulation. The major properties required for the analysis are:

For Mild Steel

- Modulus of Elasticity -  $210 \times 10^9 \text{ N/m}^2$
- Poisson's Ratio - 0.303
- Density -  $7850 \text{ Kg/m}^3$

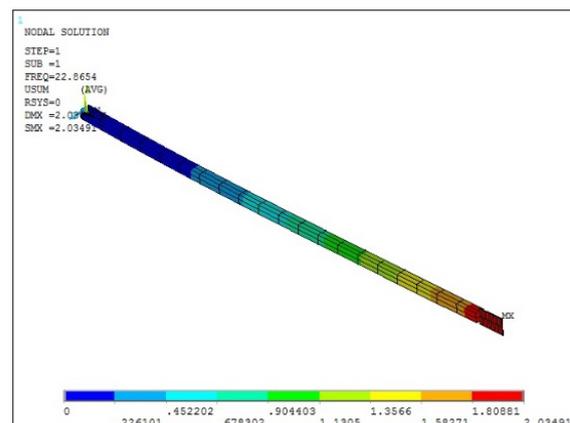
For Stainless Steel

- Modulus of Elasticity -  $200 \times 10^9 \text{ N/m}^2$
- Poisson's Ratio - 0.305
- Density -  $7700 \text{ Kg/m}^3$

## 3. Analysis using Ansys APDL 15.0

The analysis using Ansys is done to obtain the frequencies in which the non-linearity is occurring. The main focus is on the first three modes. 4cm and 15cm thin beam were used for the modal analysis and results are as shown below:

### 3.1. Modal Analysis of 4cm Thin Beam



**Fig. 3.1 : First Mode of 4cm thin beam**

The first mode obtained for the cantilever beam of length 0.7m attached to 4cm thin beam which is shown in Fig 3.1.

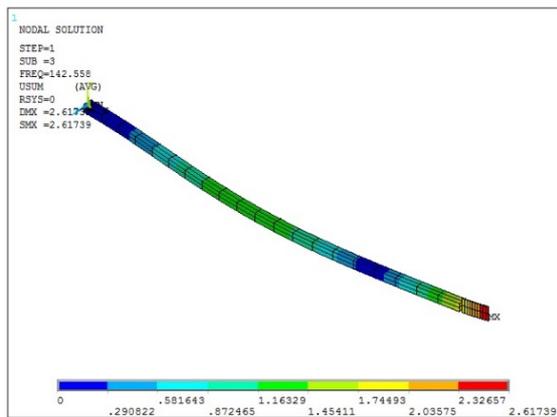


Fig. 3.2 : Second Mode of 4cm thin beam

The second mode obtained for the cantilever beam of length 0.7m attached to 4cm thin beam which is shown in Fig 3.2.

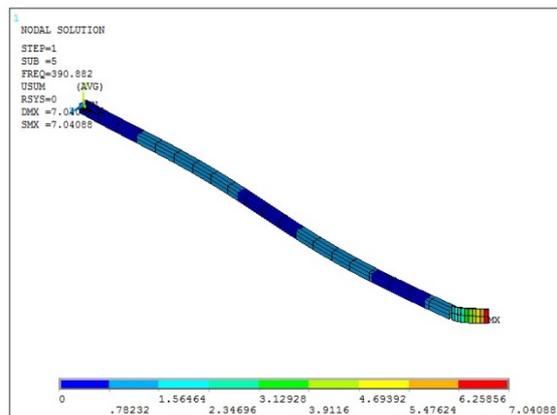


Fig. 3.3 : Third Mode of 4cm thin beam

The third mode obtained for the cantilever beam of length 0.7m attached to 4cm thin beam which is shown in Fig 3.3.

### 3.2. Modal Analysis of 15cm Thin Beam

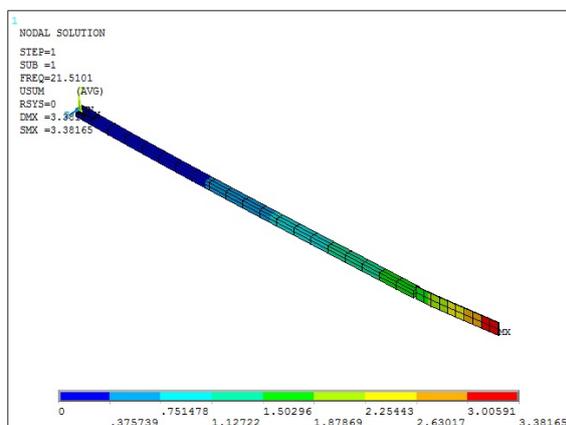


Fig. 3.1 : First Mode of 15cm thin beam

The first mode obtained for the cantilever beam of length 0.7m attached to 15cm thin beam which is shown in Fig 3.4.

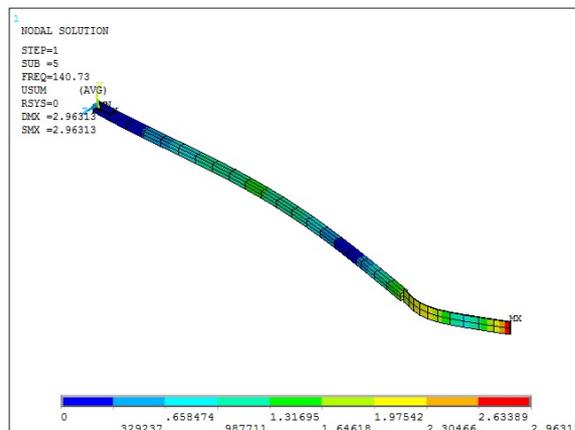


Fig. 3.2 : Second Mode of 15cm thin beam

The second mode obtained for the cantilever beam of length 0.7m attached to 15cm thin beam which is shown in Fig 3.5.

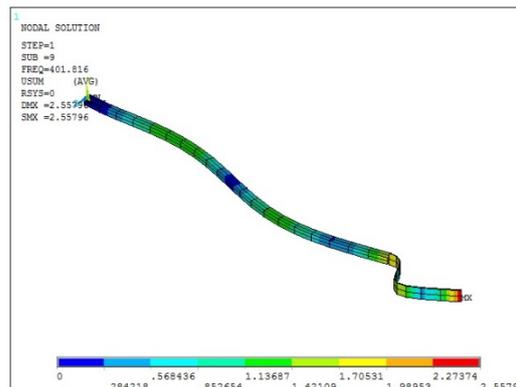


Fig. 3.3 : Third Mode of 15cm thin beam

The third mode obtained for the cantilever beam of length 0.7m attached to 15cm thin beam which is shown in Fig 3.6.

Table 3.1 : Frequencies obtained compared with reference value

SI No.	Natural Frequencies		
	Reference Paper Value	Obtained Value for 15cm Beam	Obtained Value for 4cm Beam
1	26	21.51	22.86
2	145	140.73	142.56
3	395	401.82	390.88

#### 4. Result and Dicussions

In Ansys the static analysis is done for the small displacement and the large displacement and if there is any variation in it shows that there is presence of the non-linearity. Non linearity is observed and load level is obtained by using static analysis is done in the Ansys. In thin beam of having length 4cm the non-linearity starts at load 11.2N as shown in fig 4.1 and from the reference paper [4] the value obtained by them while numerical calculation is 11.7279N.

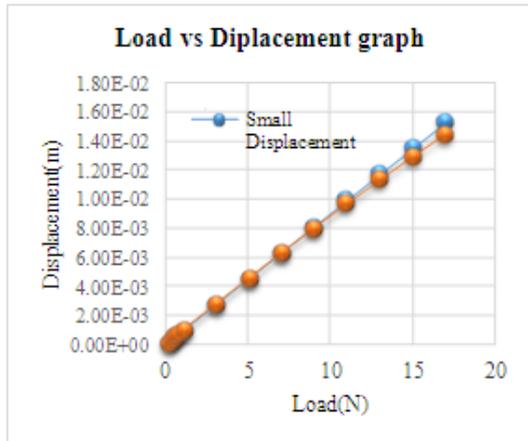


Fig. 4.1 : Prediction of load level in 4cm thin beam

The table 4.1 shows the load values obtained for the reference paper and obtained value of load in Ansys.

Table 4.1 : Load values obtained

Sl No.	Reference Paper Load(N)	Obtained Value of Load(N)
1	11.7279	11.2

From this we can understand that similarly we can predict the load for the 15cm thin beam as shown below in fig 4.2.

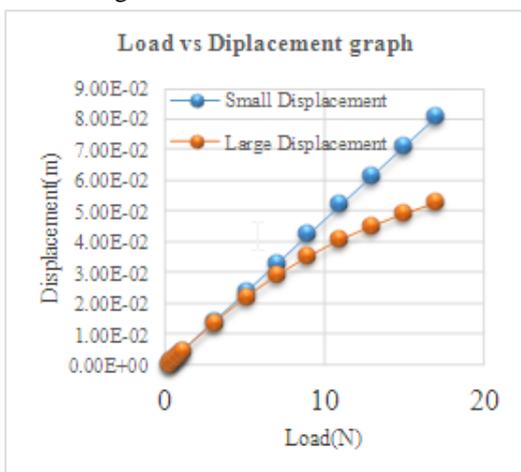


Fig. 4.2 : Prediction of load level in 15cm thin beam

#### 5. Conclusions

The cantilever system used have two beams which are connected to each other. Hence in this system the resonance condition is the cause of the nonlinearity. Both the beam interacts together to reach the resonance condition. Hence maximum deflection is obtained and lead to nonlinearity. The first modes of the both beam have excited at same time to get the interaction. The actual load obtained was a little high because the thin beam used has higher width. The beams have to deflect and the geometric nonlinearity come into effect when the thin beam deflect much more due to the resonance created. The static linear analysis of the cantilever beam have been done and the load level is compared with the load value of the reference paper [4]. Thus the load value at which the nonlinearity is initiating for a specific system is identified. Thus it could be infered that this technique can be applied to any cantilever structure and the load at which nonlinearity excited can be predicted.

#### 6. Acknowledgements

I express my sincere gratitude to Mrs. Ruby Maria Syriac, Assistant Professor, Department of Mechanical Engineering, Mar Baselios College of Engineering and Technology, Trivandrum for his invaluable guidance, advice and constant encouragement throughout the course of the thesis. I also thank Mr. Akash Rajan, PhD Student, College of Engineering, Trivandrum for his supervision, assistance and helpful suggestions given throughout the course of this work without which this work would not have been successfully completed.

I thank The Dean (PG Studies), The Principal, The Director, The Head of the Department and Dr. Y V K Sadasiva Rao, PG Coordinator, Department of Mechanical Engineering, Mar Baselios College of Engineering and Technology, Trivandrum for providing a platform for my thesis work.

#### 7. References

- [1] W. Lacarbonara, G. Rega and A. Nayfeh, "Resonant non-linear normal modes. Part I: analytical treatment for structural one-dimensional systems," International Journal of Non-Linear Mechanics, no. 38, p. 851 – 872, 2003.
- [2] C. E. Mazzilli, M. E. Soares and O. G. B. Neto, "Non-linear normal modes of a simply supported beam: continuous system and finite-element models," Computers and Structures, no. 82, p. 2683–2691, 2004.
- [3] X. Lia, J. Jib and C. H. Hansenb, "Non-linear normal modes and their bifurcation of a two DOF system with quadratic and cubic non-linearity," International Journal of Non-Linear Mechanics, no. 41, p. 1028 – 1038, 2006.
- [4] M. Peeters, G. Kerschen and J.C. Golinval, "Modal testing of nonlinear vibrating structures based on nonlinear normal modes: Experimental demonstration," Mechanical

Systems and Signal Processing, no. 25, pp. 1227-1247, 2011.

[5] L. Renson, J. Noel, G. Kerschen and A. Newerla, Nonlinear modal analysis of the smallsat spacecraft, Belgium, 2012.

[6] G. Kerschen, M. Peeters, J. C. Golinval and C. Stéphan, "Nonlinear Modal Analysis of a Full-Scale Aircraft," Journal of aircraft, 2013.

[7] N. V. Perepelkin, Y. V. Mikhlin and Christophe Pierre, "Non-linear normal forced vibration modes in systems with internal resonance," International Journal of Non-Linear Mechanics, no. 57, p. 102–115, 2013.

[8] P. Casini, O. Giannini and F. Vestroni, "Effect of damping on the nonlinear modal characteristics of a piecewise-smooth system through harmonic forced response," Mechanical Systems and Signal Processing, no. 36, p. 540–548, 2013.

[9] E. Moussi, S. Bellizzi, B. Cochelin and I. Nistor, "Nonlinear normal modes of a two degrees-of-freedom piecewise linear system," Mechanical Systems and Signal Processing, no. 64-65, p. 266–281, 2015.

[10] E. Gavassoni, P. B. Gonçalves and O. D. M. Roehl, "Nonlinear vibration modes of an off shore articulated tower," Ocean Engineering, no. 109, p. 226–242, 2015.