Imperial Journal of Interdisciplinary Research (IJIR)
Vol-2, Issue-9, 2016
ISSN: 2454-1362, http://www.onlinejournal.in

Review paper on a New Approach to Design Antenna Arrays Using Polyfractal Arrays

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Abstract: *Polyfractal arrays are subset of fractal-random arrays, which combines the advantages of both periodic and random arrays together. In this paper we are reviewing new design methodologies that utilize nature-inspired design techniques to optimize array layouts that combine both ordered and periodic array properties in their geometries. These arrays are called polyfractal arrays, provides low sidelobe levels with wider bandwidth and no grating lobes with narrow beam width when optimized with different type of nature inspired techniques called Genetic algorithm. This paper also demonstrates how self similar properties of polyfractal arrays increase the speed of array factor calculations. We will discuss several examples also which will show how polyfractal arrays are better than conventional arrays and comparison of advantages and disadvantages of different type of optimization techniques used for design of polyfractal arrays with their array factor radiation pattern calculation is also discussed here.*

1. Introduction

The antenna is one of the most important equipment which is used in radar and wireless communication systems. Mainly antennas are characterized by several terms like directivity, Gain, sidelobe levels, bandwidth, radiation pattern and its efficiency. The two common types of antenna arrays layouts are Periodic array and random array. Periodic arrays possess many good characteristics; they typically have low side-lobe levels and easily definable antenna positions. However, Periodic arrays operating bandwidth is extremely narrow so after certain bandwidth large grating lobes will appear in the radiation pattern. On the other hand, Random arrays have many degrees of freedom in their designs, but random arrays are not as effective in suppressing side-lobe levels as periodic arrays and in defining the positions of antenna elements. That is why random arrays are not ideal for many applications.
In 1960's aperiodic arrays are used to improve antenna array bandwidth which possess ordered or Optimized geometries [1-3].here it is shown that the aperiodic array layouts have relatively low sidelobes levels over a large bandwidth in comparison to periodic array.
After that, In 1983 Mandelbrot [4] introduced the term inspired by nature like tree, leaves, ferns, coastlines etc., called fractal, which means to break, to describe a family of complex shapes that possess self-similarity properties in their geometrical structure. The main properties of fractal arrays are frequency independent, multi-band and approach to thinning with rapid beam forming algorithms.

In fractals are used in many specialization of engineering and science in which fractal electrodynamics [5-9] combines fractals with antenna theory.

First time, Fractal term in the field of antenna theory is used by Kim and Jaggard [10] for design of low sidelobe level arrays based on theory of random fractals. Fractals can be of two types either deterministic fractal arrays or random fractal arrays.

Fractal random arrays combine beneficial properties of both periodic and random arrays. Fractal random arrays are constructed using randomly selected generators from a set of probable choices so the number of possible fractal random array combinations can become so large that it will be so difficult to recreate the arrays from small set of parameters and fractal random arrays are not recursive due to which rapid beamforming algorithm cannot be developed for efficient calculation of their radiation patterns. Due to these drawbacks of fractal random arrays we go for polyfractal arrays. Fractal random arrays and polyfractal arrays can be constructed by using iterated function system (IFS) [11] method based on series of affine linear transformations. Polyfractal arrays are a subset of fractal-random arrays which perform similarly to fractal random arrays. So in this article we will review the different type of nature inspired optimization techniques like GA [12-14] and PSO [15-17] to find out optimal layout for polyfractal array with recursive beamforming algorithm which results in low sidelobe levels and wider bandwidth with no grating lobes and narrow beamwidth. This paper also discusses how self-similar properties of

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polyfractal arrays increase the speed of array factor calculations.

2. Polyfractal array

Polyfractal array [18] is a special type of subset of fractal random array which overcomes from limitations of fractal-random arrays. Both fractal-random arrays and polyfractal arrays are constructed from multiple generators; however, the generators associated with polyfractal arrays consist of an additional parameter included in the description of every branch or affine, called *connection factors*, which allow polyfractal arrays to be constructed recursively by dictating which generator is applied to the end of a particular branch.

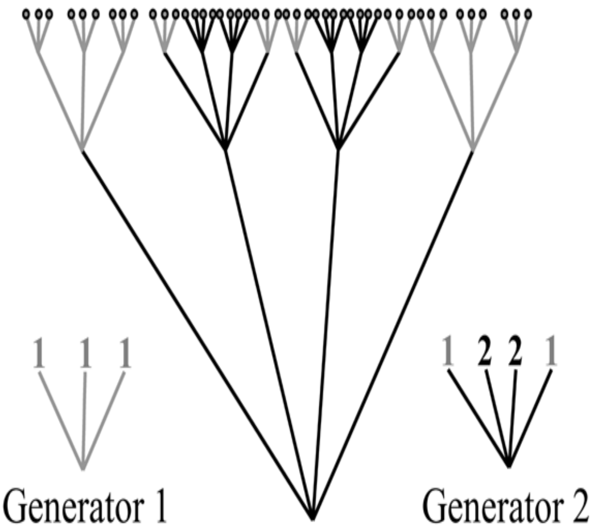

Fig.1: Construction of a 46-element polyfractal array using connection factors.

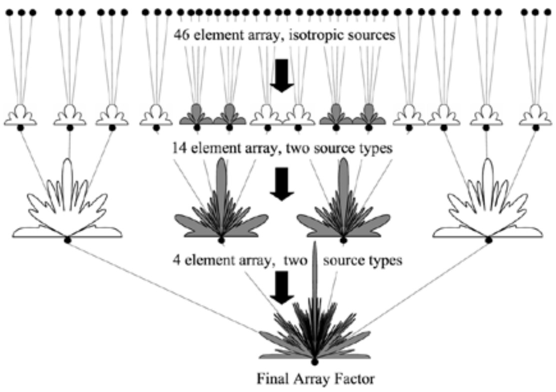

Fig.2: Rapid recursive beamforming algorithm for a two generator polyfractal array.

In fig.1, the branches ending with number 1 have generator 1 connected to their ends. Similarly, the branches ending with number 2 have generator 2 connected to their ends. The ends of the topmost branches represent the positions of the antenna elements. So it indicates the polyfractal arrays are recursive in nature and can be exactly reconstructed from a small set of parameters.

IFS method is used to construct polyfractal array, based on affine linear transformation. The advance similitude representation of affine linear transformation for polyfractal array is given as

$$w_{m,n}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix} s_f\cos(\varphi_{m,n}+\Psi_{m,n}) & -s_f\sin(\varphi_{m,n}+\Psi_{m,n}) \\ s_f\sin(\varphi_{m,n}+\Psi_{m,n}) & s_f\cos(\varphi_{m,n}+\Psi_{m,n}) \end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}\rho_{m,n}\cos(\varphi_{m,n})\\ \rho_{m,n}\sin(\varphi_{m,n})\end{pmatrix}$$

Where $\rho_{m,n}$, $\varphi_{m,n}$, $\Psi_{m,n}$ are three local parameters and S_f is the fractal scale parameter.

Polyfractal arrays are constructed from multiple generators, 1, 2, 3 ...*M*, each of which has a corresponding Hutchinson operator *W*1, *W*2 ...*Wm*.It has 3 local parameters along with a fourth parameter called connection factors which has a integer value between 1to M, number of generators used to construct polyfractal array.

To construct *l+1* stage polyfractal arrays from set of possible stage *l*, Hutchinson operator is used.

$$F_{l+1,m} = W_m \left(\{F_{l,1},F_{l,2},........,F_{l,m}\}\right) = \left[\bigcup_{n=1}^{N_m} w_{m,n}\left(F_{l,k_{m,n}}\right)\right]$$

Where *W* is known as Hutchinson operator and F_l is a fractal of stage *l*.

Each affine linear transformation, $w_{m,n}$, can only be performed on stage polyfractal arrays where the generator applied at stage matches the connection factor, $k_{m,n}$.The connection factors provides unique polyfractal array geometry associated with each Hutchinson operator.

final polyfractal array $F_{L,P}$ is represented by

$$F_{L,P} = s_g W_P \left(\{F_{L-1,1},F_{L-1,2},........,F_{L-1,M}\}\right)$$

$$= s_g\left[\bigcup_{n=0}^{N_P} w_{m,n}\left(F_{L-1,k_{P,n}}\right)\right]$$

Here P is a global connection factor with S_g, global scale parameter. These are the initial global scale parameters used to scale the array for minimum spacing requirement.

As polyfractal arrays have recursive in nature. So, Recursive beamforming algorithms for polyfractal arrays is given as expression of the stage l polyfractal subarray radiation pattern

$$FR^L_{l,m}(\theta,\varphi) = I_o\sum_{n=1}^{Nm} FR^L_{l-1,k_{m,n}}\left(\theta,\varphi-\varphi_{m,n}-\psi_{m,n}\right) * \exp j\left[k\left(s_g\left(s_f\right)^{L-l} r_{m,n}\right)\sin\theta\cos\left(\varphi-\varphi_{m,n}\right)\right]$$

The overall radiation pattern can be determined by using isotropic sources for the initial subarray radiation patterns and recursively applying the expression until the final,L stage radiation pattern is obtained.

3. Techniques used to optimize polyfractal array

The term polyfractal array was introduced by J.S. Petko and D.H. Werner [18] in 2005, he discussed a type of nature-based design process that applies a specially formulated genetic algorithm optimization technique to find optimized polyfractal array layouts. In this specially formulated genetic algorithm, an initial population of polyfractal arrays are randomly created up to L stages of the fractal tree using 1,2....,*M* generators. In that First, two arrays are randomly selected from the population and paired for genetic crossover. However, the chromosomes for polyfractal arrays are not necessarily the same size. To overcome this problem, the parent chromosomes of the fractal-random array are broken apart. Crossover is performed on the global parameters of the arrays, but the generators from each chromosome are paired again for second crossover, the generator crossover, to be performed on them. Now also, if numbers of branches of each generator are different, then generators are also broken apart for a third crossover, called branch crossover. In the third crossover case, the branches from each generator are paired the branches are then grouped into the new generators and creates the new global parameters. The final result is two offspring chromosomes. A mutation in a genetic algorithm is the random changing of a parameter to an arbitrary value. The final operation of the genetic algorithm is natural selection. In addition, he discussed how the rapid beamforming algorithm can be used to reduce the time required for the genetic algorithm to find radiation pattern with lower sidelobe levels and smaller beam widths. A complete process of crossovers, mutations, and natural selection is referred to as generation of the genetic algorithm. The main advantage of polyfractal arrays provides a simple and compact way to describe complicated structures using only a few parameters. Second, a rapid beamforming algorithm for faster calculation of radiation patterns of polyfractal arrays.

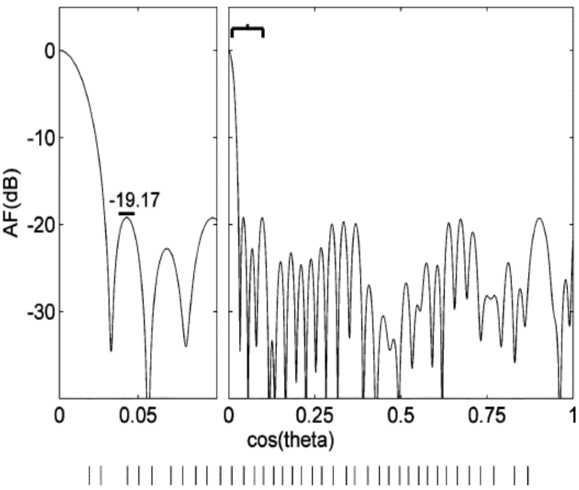

Fig.3: 46 element genetically optimized polyfractal with its radiation pattern

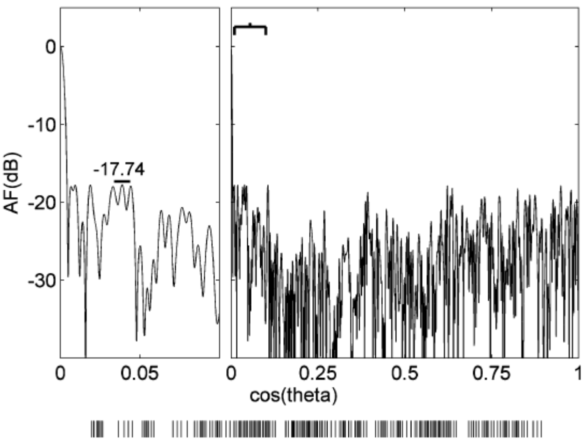

Fig.4: 178 element genetically optimized polyfractal with its radiation pattern.

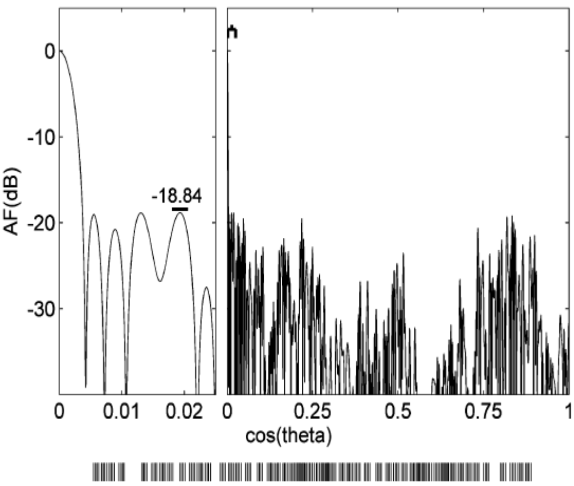

Fig.5: 256 element genetically optimized polyfractal with its radiation pattern

In paper [18] 46, 178 and 256 genetically optimized polyfractal arrays are designed whose sidelobe levels are -19.17dB, -17.74dB and –18.84dB with 1.57, 0.234 and 0.203 degree beamwidth respectively. So it is concluded that as polyfractal arrays have recursive property nature. Due to that, it is found that using recursive beamforming algorithm speed of array factor calculation is faster than conventional beamforming algorithm which makes GA to converge faster and genetically optimized polyfractal arrays have low side-lobe levels and small beamwidths.

Polyfractal arrays always have unique structures, using this unique property, a new technique is used for complex design of optimal Large N-array polyfractal array using Generator autopolyploidy [19] technique based on genome autopolyploidy[32]. Generator autopolyploidy is a process that doubles the number of generator for a polyfractal array.

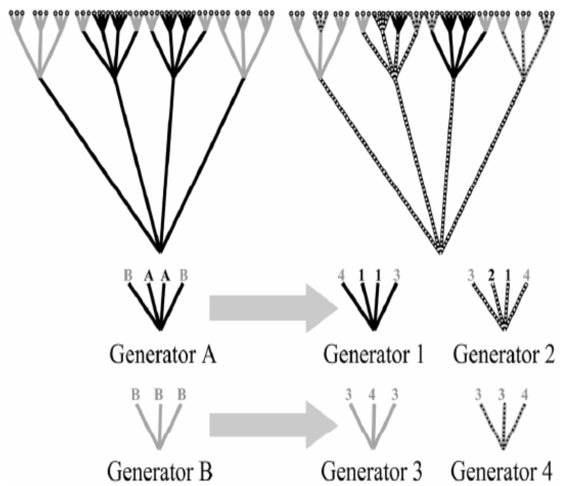

Fig.6: generator- autopolyploidy process for2 polyfractal generators.

In Generator autopolyploidization first, each polyfractal generator is divided into two identical parts similar to the original. Second, the connection factors used to select previous generator are uniformly divided to choose between the two new duplicates so that the arrays should be same but number of parameters will be twice.

The main benefit of generator autopolyploidy is to evolve large N-array complex designs for polyfractal array layouts.

Autopolyploidy is used for efficient complex design of optimal polyfractal array and is used when GA optimization provides premature convergence.

Other than geometrical advantages of polyfractals, generator- autopolyploidy based chromosome increase the efficiency and reduces the convergence time of the GA [25], [26]. And autopolyploidy process improves the polyfractal array designs also.

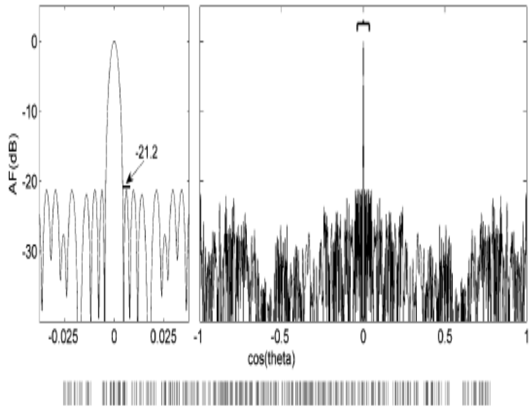

Fig.7: 256 element optimized polyfractal array using generator- autopolyploidy process and its radiation pattern.

Now if we compare fig.5 with fig.7, it is found that generator-autopolyploidy process based optimized polyfractal array provides more reduced sidelobes rather than specially formulated GA optimized polyfractal array.

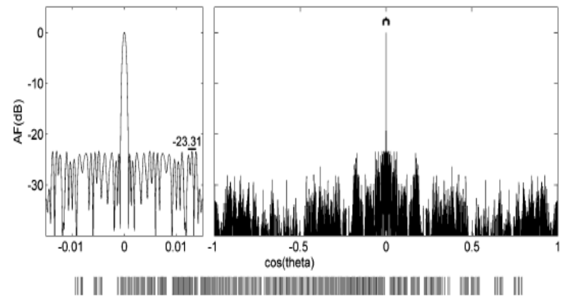

Fig.8:1406 element optimized polyfractal array using generator- autopolyploidy process and its radiation pattern.

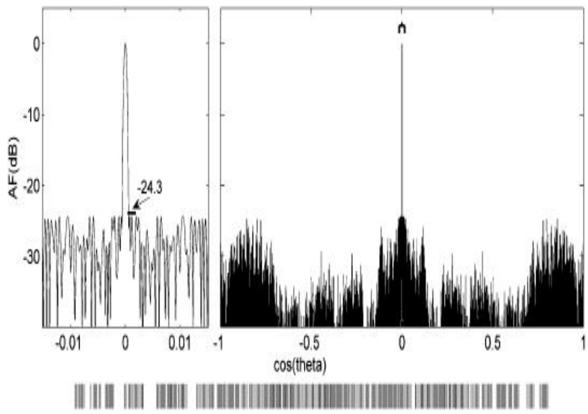

Fig.9:1616 element optimized polyfractal array using generator- autopolyploidy process and its radiation pattern.

Fig. 8 and 9 shows the design of 1406 and 1616 elements optimized polyfractal array using autopolyploidy process with -23.31dB and -24.3dB sidelobe levels respectively with faster calculation of radiation pattern. Generator autopolyploidization process will used in GA when the fitness of fittest member in the population doesn't improve over 30 generations.

After applying generator autopolyploidy process the time required to calculate the radiation pattern using recursive beamforming algorithm increases as compare to conventional radiation pattern calculation.

Techniques used in [18,19] are for optimization of a single objective function while for some of the real world applications needs a type of antenna arrays with large number of elements which provides low side lobes levels with wide bandwidth and faster radiation pattern calculations. So, multi objective optimization techniques are used to design this type of antenna arrays.

In 2008, J.S. Petko and D.H. Werner [20] studied many pareto front algorithms [21-28] and presented a new multi objective optimization technique called strength pareto optimization algorithm (SPEA) [24,25], this optimization technique was used with generator autopolyploidy technique[19] to design optimized large-N polyfractal array layout which provides lower sidelobes levels simultaneously at several frequencies.

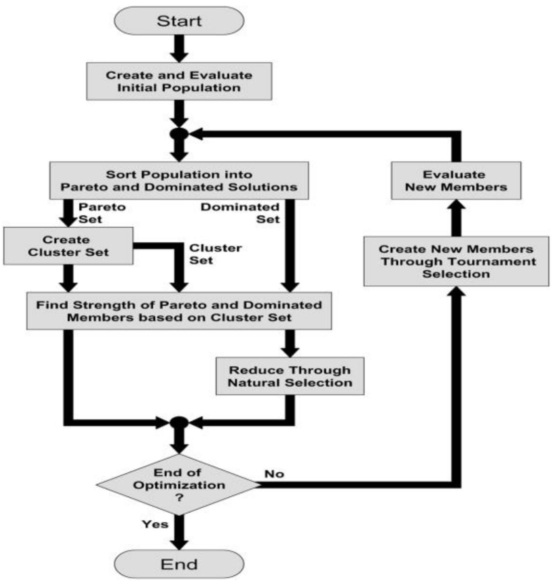

Fig.10: SPEA flowchart.

Using this SPEA technique a pareto front [20] is created which will provide the most efficient set of solution from the initial population according to our optimization parameters and no other solution dominates over these solution.

SPEA is algorithm used to create ultra wideband polyfractal arrays. So here it is found that using SPEA, optimized polyfractal arrays provided an ultra wide bandwidth with no grating lobes.

In [20] several examples are discussed in which polyfractal arrays layout are designed using pareto strength algorithm which provides low sidelobes levels over two different frequencies(0.5λ and 3.0λ)

Below table shows the side level for different number of antenna array elements over different several frequencies.

No. of elements	SLL	HPBW	Average spacing	SLL	HPBW	Average spacing
1924	-19.12dB	0.027°	1.05 λ	-19.08dB	0.0045°	6.32 λ

So from table it is observed that polyfractal arrays designed using SPEA techniques consist of lower sidelobe levels with no grating lobes over ultra wide bandwidths. Here a uniformly 1924-element array has -15.97dB sidelobe level over more than a 40:1 bandwidth.

Now according to over modern communications and remote sensing applications we need an antenna arrays which have broadband, multi-band, multi-frequency operation together in one design. So to fulfil these requirements, a technique, called interleaved polyfractal array based on GA [29] developed for the design of large-aperture ultra wideband array antenna systems that possess multiple main beams, in which each antenna has different frequencies and independent steering angle. The main goal of these design layouts are to keep low sidelobe levels and suppressed grating lobes over UWB. In paper [29] discussed many layout of interleaved polyfractal array, out of which one of the interleaved polyfractal array is constructed using two 1959-element polyfractal arrays which has 3918 antenna elements system and the spacing between each element is scaled to 1.0λ and 0.5λ for entire system

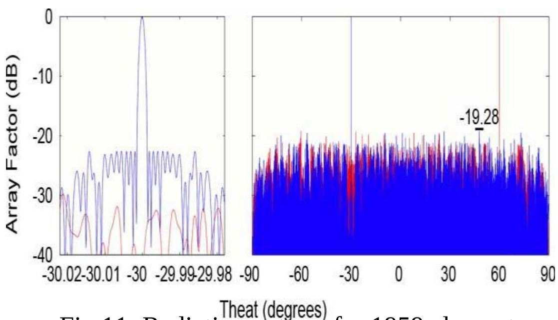

Fig.11: Radiation pattern for 1959 elements optimized polyfractal array.

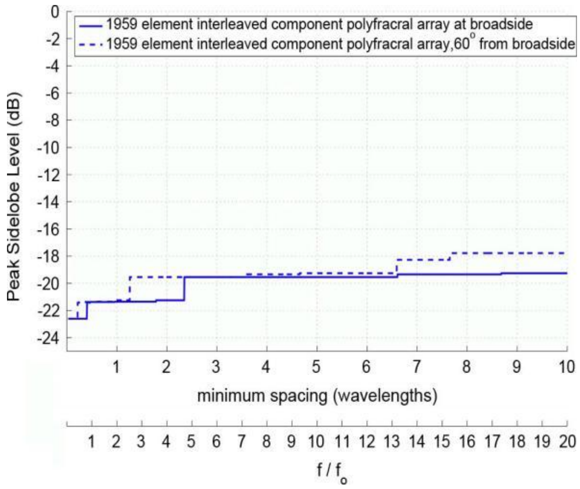

Fig.12: Sidelobe level performance of 3918-element interleaved polyfractal array system.

Fig.11 shows the two radiation patterns of the interleaved antenna array system. Here one radiation pattern is steered -30 degree from broadside while other is steered +60 degree from broadside. In fig.12 sidelobe levels are presented where dark blue line represents broadside operation while blue dashed line represents the operation of an array steered 60 from broadside.

It is found that 3918-element interleaved Polyfractal array have -19.28dB sidelobe levels with no grating lobes steered independently up to 60 degree from broadside and provided 20:1 Bandwidth.

After design of optimized interleaved polyfractal array layouts F.Namin, J.S. Petko and D.H. Werner applied concept of polyfractal arrays to design a practical application called micro UAV swarm based antenna arrays. In 2012 they applied the concept of planer polyfractal array and aperiodic tiling arrays theory to design optimal micro UAV swarm based antenna arrays [30] where these both concepts can work over wide bandwidths and inter elements spacing. The concept of creating planer polyfractal array is similar to concept of linear polyfractal array. The main advantage to use aperiodic arrays over periodic arrays is use of less number of elements in aperiodic arrays than similar size of periodic arrays and low sidelobe levels over wide bandwidths

As the aperiodic micro UAV swarms have a disadvantage to easily effect by turbulences and position errors [31] to remove these effects a phase-compensation algorithm [32] is used which provides coherent beam radiation patterns.

Figure 13 shows the layout of aperiodic micro UAV swarm based on 319 element polyfractal antenna array. This 319 element planer polyfratal antenna array is constructed by 3 polyfractal generators. Micro UAV swarm can easily be affected by noise in nature so fig.14 shows the noise affected planer polyfractal array. A phase compensation theorem [30, 31] is used to remove this noise and positional errors.

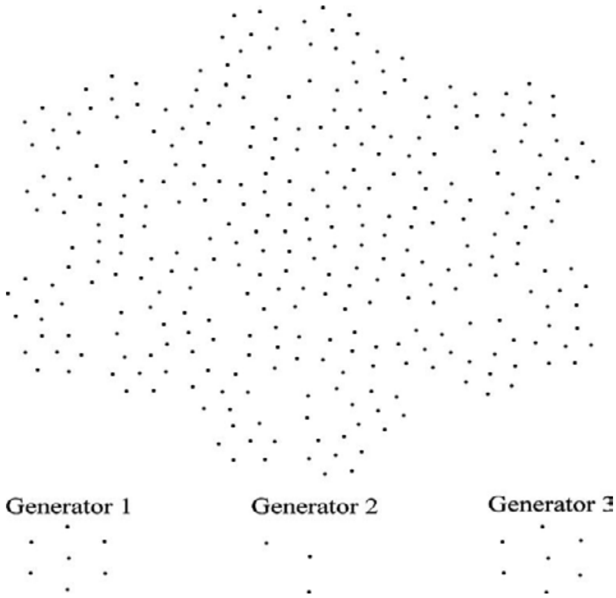

Fig.13: Construction of planer 319 elements optimized polyfractal array using 3 generators.

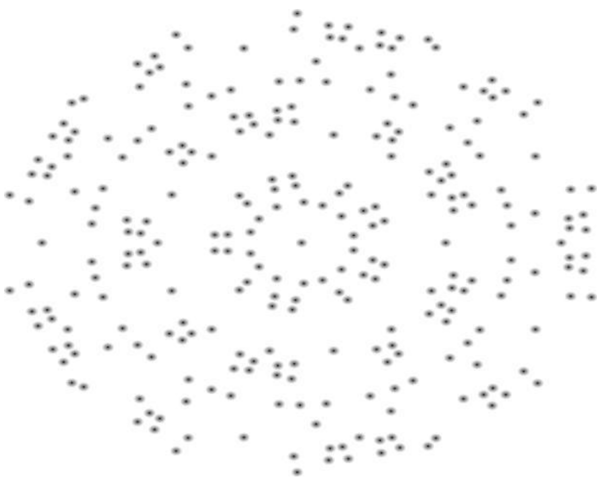

Fig.14: 319 elements of polyfractal array effected by Gaussian noise σ=0.1λ.

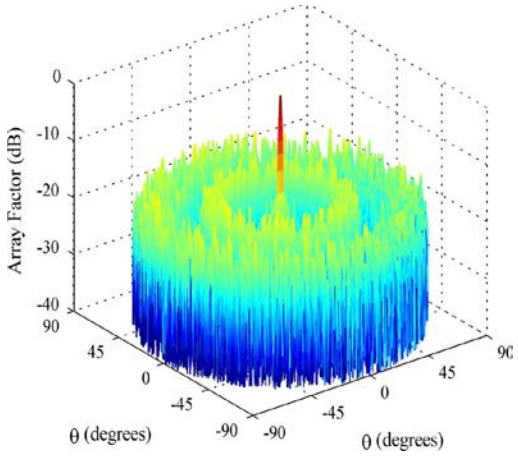

Fig.15: Normalized array factor for the optimized polyfractal array with 319 elements.

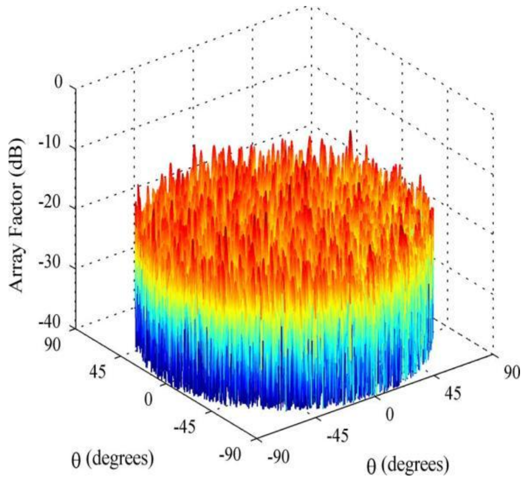

Fig.16: The corrupted radiation pattern for a micro-UAV swarm based on the 319 element optimized polyfractal array.

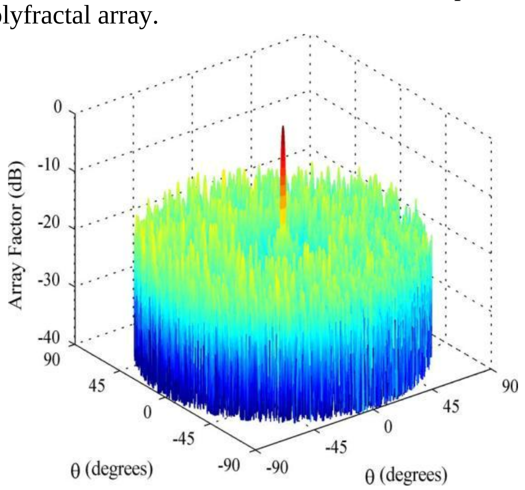

Fig.17: The phase corrected radiation pattern for a micro-UAV swarm based on the 319 element optimized polyfractal array.

Fig.16 shows the noise corrupted radiation pattern calculation for optimized 319 elements and it is found that small position error also effect main beam in radiation pattern.

Fig17 explains about radiation pattern of 319 elements polyfractal array after removal of noise and positional array using phase compensation algorithm. Here it is found that the sidelobe level after using phase compensation algorithm is -14dB which is equal to the side lobe level of the normalized radiation pattern of optimized polyfractal array.

4. Conclusion

In this review paper we have discussed a special type of nature based optimization algorithm called genetic algorithm to design a special type of antenna array called polyfractal array, which is a subset of fractal random array. The crossover and mutation process in genetic algorithm used here are formulated according to the parameters. Generator autopolyploidy process is used to avoid GA premature convergence and to design large N-array polyfractal array which provide efficient faster calculation of radiation pattern. Autopolyploidy and SPEA technique together is used for multiple objective optimization design of large N-array polyfractal array which provide low sidelobe levels at different several frequencies with no grating lobes. After using these two techniques another technique called interleaving is used on large N-array optimized polyfractal array over UWB. Then a micro UAV swarm antenna array is designed based on planer poyfractal array. Here in this article we have discussed many examples which shows that how polyfractal array provide faster calculation of radiation pattern using recursive beamforming algorithm than conventional DFT array factor calculation.
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5. References

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