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On Intuitionistic Fuzzy Generalized γ open sets

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Abstract: *In this paper, we have introduced intuitionistic fuzzy generalized γ open sets, and investigated some of their basic properties and obtained some interesting results.*

Keywords: *Intuitionistic fuzzy topology, Intuitionistic fuzzy γ open sets, Intuitionistic fuzzy generalized γ open sets.*

1. Introduction

The notion of intuitionistic fuzzy sets was introduced by Atanassov[1] as a generalization of fuzzy sets. In 1997, Coker[2] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we have introduced intuitionistic fuzzy generalized γ open sets, and investigated some of their basic properties and obtained some interesting results.

2. Preliminaries

Definition 2.1[1]**:** An *intuitionistic fuzzy set*(IFS in short) A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the function $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2[1]**:** Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

a) $A \subseteq B$ in and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$;
b) $A = B$ in and only if $A \subseteq B$ and $A \supseteq B$;
c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$;
d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$;
e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0\sim = \langle x, 0, 1\rangle$ and $1\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3[2]**:** An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFS, in X satisfying the following axioms:

(i) $0\sim, 1\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$

In this case the pair (X, τ) is called *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4[4]**:** An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

(i) *intuitionistic fuzzy γ closed set* (IFγCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$

(ii) *intuitionistic fuzzy γ open set* (IFγOS in short) if $A \subseteq int(cl(A)) \cup cl(int(A))$

Definition 2.5[4]**:** Let A be an IFS in an IFTS (X, τ). Then the γ-interior and γ-closure of A are defined as

$\gamma int(A) = \cup \{G / G$ is an IFγOS in X and $G \subseteq A\}$

$\gamma cl(A) = \cap \{K / K$ is an IFγCS in X and $A \subseteq K\}$

Note that for any IFS A in (X, τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A)^c = (\gamma cl(A))^c$.

Result 2.6[5]: Let A be an IFS in (X, τ), then

(i) $\gamma cl(A) \supseteq A \cup cl(int(A)) \cap int(cl(A))$

(ii) $\gamma int(A) \subseteq A \cap cl(int(A)) \cap int(cl(A)$

Definition 2.7[5]**:** An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy generalized γ closed set* (IFGγCS for short) if $\gamma cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ). The family of all IFGγCSs of an IFTS (X, τ) is denoted by IFGγC(X).

3. Intuitionistic Fuzzy Generalized γ open sets

In this section we have introduced intuitionistic fuzzy generalized γ open sets and studied some of their properties. We have provided some of the characterization.

Definition 3.1: The complement A^c of an IFGγCS A in an IFTS (X, τ) is called an *intuitionistic fuzzy generalized γ open set* (IFGγOS in short) in X.

The family of all IFGγOSs of an IFTS (X, τ) is denoted by IFGγO(X).

Example 3.2: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then the IFS $A = \langle x, (0.5_a, 0.8_b), (0.4_a, 0.2_b)\rangle$ is an IFS in X. Then IFγC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. Then A is an IFGγOS, as its complement $A^c = \langle x, (0.4_a, 0.2_b), (0.5_a, 0.8_b)\rangle$ is an IFGγCS. We have $A^c \subseteq G_1$. Now $\gamma cl(A^c) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 \subseteq G_1$. Therefore A^c is an IFGγCS in X and hence A is an IFGγOS.

Theorem 3.3: Every IFOS, IFROS[6], IFPOS[3], IFSOS[3], IFαOS[3], IFGOS[7] and IFγOS[4] but not conversely.

Proof: Straight forward.

Example 3.4: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then the IFS $A = \langle x, (0.5_a, 0.8_b), (0.4_a, 0.2_b)\rangle$ is an IFS in X. Then IFγC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. Then A is an IFGγOS, as its complement $A^c = \langle x, (0.4_a, 0.2_b), (0.5_a, 0.8_b)\rangle$ is an IFGγCS. We have $A^c \subseteq G_1$. Now $\gamma cl(A^c) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 \subseteq G_1$. This implies that A^c is an IFGγCS in X and hence A is an IFGγOS. Now since int(A) = $G_1 \neq A$. Therefore A is not an IFOS in X.

Example 3.5: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then the IFS $A = \langle x, (0.5_a, 0.8_b), (0.4_a, 0.2_b)\rangle$ is an IFS in X. Then IFγC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. Then A is an IFGγOS as its complement $A^c = \langle x, (0.4_a, 0.2_b), (0.5_a, 0.8_b)\rangle$ is an IFGγCS. We have $A^c \subseteq G_1$. Now $\gamma cl(A^c) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 \subseteq G_1$. This implies that A^c is an IFGγCS in X and hence A is an IFGγOS. Now since $int(cl(A)) = int(G_1^c) = G_1 \neq A$. Therefore A is not an IFROS in X.

Example 3.6: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then the IFS $A = \langle x, (0.5_a, 0.8_b), (0.4_a, 0.2_b)\rangle$ is an IFS in X. Then IFγC(X) = {0~, 1~, $\mu_a \in [0, 1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. Then A is an IFGγOS as its complement $A^c = \langle x, (0.4_a, 0.2_b), (0.5_a, 0.8_b)\rangle$ is an IFGγCS. We have $A^c \subseteq G_1$. Now $\gamma cl(A^c) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 \subseteq G_1$. This implies that A^c is an IFGγCS in X and hence A is an IFGγOS. Now since $A \not\subseteq int(cl(A)) = int(G_1^c) \not\subseteq G_1$. Therefore A is not an IFPOS in X.

Example 3.7: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then the IFS $A = \langle x, (0.5_a, 0.8_b), (0.4_a, 0.2_b)\rangle$ is an IFS in X. Then IFγC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. Then A is an IFGγOS as its complement $A^c = \langle x, (0.4_a, 0.2_b), (0.5_a, 0.8_b)\rangle$ is an IFGγCS. We have $A^c \subseteq G_1$. Now $\gamma cl(A^c) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 \subseteq G_1$. This implies that A^c is an IFGγCS in X and hence A is an IFGγOS. Now since $A \not\subseteq cl(int(A)) = cl(G_1) \not\subseteq G_1^c$. Therefore A is not an IFSOS in X.

Example 3.8: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b)\rangle$. Then the IFS $A = \langle x, (0.5_a, 0.8_b), (0.4_a, 0.2_b)\rangle$ is an IFS in X. Then IFγC(X) = {0~, 1~, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / either $0.3 \leq \mu_b < 0.2$ whenever $\mu_a \geq 0.5$ (or) $\mu_a < 0.4$ and $0 < \mu_b + \nu_b \leq 1$}. Then A is an IFGγOS as its complement $A^c = \langle x, (0.4_a, 0.2_b), (0.5_a, 0.8_b)\rangle$ is an IFGγCS. We have $A^c \subseteq G_1$. Now $\gamma cl(A^c) = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 \subseteq G_1$. This implies that A^c is an IFGγCS in X and hence A is an IFGγOS. Now since $A \not\subseteq$

int(cl(int(A))) = int(cl(G_1)) = int(G_1^c) ⊈ G_1. Therefore A is not an IFαOS in X.

Example 3.9: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where G_1 = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)⟩, G_2 = ⟨x, (0.4_a,0.2_b), (0.6_a, 0.8_b)⟩. Then the IFS A = ⟨x, (0.5_a, 0.8_b), (0.4_a, 0.2_b)⟩ is an IFS in X. Then IFγC(X) = {0~, 1~, μ_a ∈ [0,1], μ_b ∈ [0,1], ν_a ∈ [0,1], ν_b ∈ [0,1] / either 0.3 ≤ μ_b < 0.2 whenever μ_a ≥ 0.5 (or) μ_a < 0.4 and 0 < μ_b + ν_b ≤ 1}. Then A is an IFGγOS as its complement A^c = ⟨x, (0.4_a, 0.2_b), (0.5_a, 0.8_b)⟩ is an IFGγCS. We have A^c ⊆ G_1. Now γcl(A^c) = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 ⊆ G_1. This implies that A^c is an IFGγCS in X and hence A is an IFGγOS. Now since G_1^c ⊈ int(A) ⊈ G_1. Therefore A is not an IFGOS in X.

Example 3.10: Let X = {a, b} and let τ = {0~, G_1, G_2, 1~} be an IFT on X, where G_1 = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)⟩, G_2 = ⟨x, (0.4_a,0.2_b), (0.6_a, 0.8_b)⟩. Then the IFS A = ⟨x, (0.5_a, 0.8_b), (0.4_a, 0.2_b)⟩ is an IFS in X. Then IFγC(X) = {0~, 1~, μ_a ∈ [0,1], μ_b ∈ [0,1], ν_a ∈ [0,1], ν_b ∈ [0,1] / either 0.3 ≤ μ_b < 0.2 whenever μ_a ≥ 0.5 (or) μ_a < 0.4 and 0 < μ_b + ν_b ≤ 1}. Then A is an IFGγOS as its complement A^c = ⟨x, (0.4_a, 0.2_b), (0.5_a, 0.8_b)⟩ is an IFGγCS. We have A^c ⊆ G_1. Now γcl(A^c) = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) = G_1 ⊆ G_1. This implies that A^c is an IFGγCS in X and hence A is an IFGγOS. Now since A ⊈ int(cl(A)) ∪ cl(int(A)) = int(G_1^c) ∪ cl(G_1) = G_1 ∪ G_1^c ⊈ G_1^c. Therefore A is not an IFγOS in X.

Remark 3.11: The union of any two IFGγOS is not an IFGγOS in general as seen in the following example.

Example 3.12: Let X = {a, b} and τ = {0~, G_1, G_2, 1~} where G_1 = ⟨x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)⟩, G_2 = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩ then the IFSs A = ⟨x, (0.5_a, 0.1_b), (0.5_a, 0.9_b)⟩ and B = ⟨x, (0.4_a, 0.3_b), (0.5_a, 0.7_b)⟩ are IFGγOSs in (X, τ). Let us prove A^c and B^c are IFGγCS. Then IFγC(X) = {0~, 1~, μ_a ϵ [0,1], μ_b ϵ [0,1], ν_a ϵ [0,1], ν_b ϵ [0,1] / μ_a < 0.5 whenever μ_b ≥ 0.6, μ_b < 0.6 whenever μ_a ≥ 0.5 and 0 ≤ μ_a + ν_a ≤ 1 and 0 ≤ μ_b + ν_b ≤ 1}. Now A^c ⊆ 1~ and γcl(A^c) = 1~ ⊆ 1~ which implies A^c is an IFGγCS in X, and hence A is an IFGγOS in X. We have B^c ⊆ 1~ and γcl(B^c) = 1~ ⊆ 1~. Therefore B^c is an IFGγCS in X and hence B is an IFGγOS in X. Now A ∪ B = ⟨x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)⟩ is not an IFGγOS, since $(A ∪ B)^c$ = ⟨x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)⟩ ⊆ G_1 but γcl$(A ∪ B)^c$ = 1~ ⊈ G_1. Therefore $(A ∪ B)^c$ is not an IFGγCS in X and hence A ∪ B is not an IFGγOS in X.

Remark 3.13: The intersection of any two IFGγOS is not an IFGγOS in general as seen from the following example.

Example 3.14: Let X = {a, b} and τ = {0~, G_1, G_2, 1~} where G_1 = ⟨x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)⟩, G_2 = ⟨x, (0.5_a, 0.5_b), (0.4_a, 0.4_b)⟩. Then the IFSs A = ⟨x, (0.4_a, 0.5_b), (0.5_a, 0.4_b)⟩ and B = ⟨x, (0.5_a, 0.2_b), (0.4_a, 0.6_b)⟩ are IFGγOSs in (X, τ). Let us prove A^c and B^c are IFGγCS. Then IFγC(X) = {0~, 1~, μ_a ϵ [0,1], μ_b ϵ [0,1], ν_a ϵ [0,1], ν_b ϵ [0,1] / μ_a < 0.5 whenever μ_b ≥ 0.5, μ_b < 0.5 whenever μ_a ≥ 0.5 and 0 ≤ μ_a + ν_a ≤ 1, and 0 ≤ μ_b + ν_b ≤1}. Now γcl(A^c) = A^c. We have A^c ⊆ G_1 and A^c ⊆ G_2 we have γcl(A^c) = A^c. Therefore γcl(A^c) ⊆ G_1 whenever A^c ⊆ G_1 and γcl(A^c) ⊆ G_2 whenever A^c ⊆ G_2, where G_1 and G_2 are IFOS in X. This implies that A^c is an IFGγCS in X. Hence A is an IFGγOS. We have B^c ⊆ G_1 and since γcl(B^c) = B^c, γcl(B^c) ⊆ G_1. Therefore B^c is an IFGγCS in X, and hence B is an IFGγOS in X. Now to prove A ∩ B = ⟨x, (0.4_a, 0.2_b), (0.5_a, 0.6_b)⟩ is an IFGγOS. Let us prove $(A ∩ B)^c$ is an IFGγCS. Now since $(A ∩ B)^c$ = ⟨x, (0.5_a, 0.6_b), (0.4_a, 0.2_b)⟩ ⊆ G_1 but γcl$(A ∩ B)^c$ = 1~ ⊈ G_1. Therefore $(A ∩ B)^c$ is not an IFGγCS in X and hence A ∩ B is not an IFGγOS in X.

Theorem 3.15: Let (X, τ) be an IFTS. Then for every A ∈ IFGγO(X) and for every B ∈ IFS(X), γint(A) ⊆ B ⊆ A ⇒ B ∈ IFGγO(X).

Proof: Let A be an IFGγOS of X and B be any IFS of X. Let γint(A) ⊆ B ⊆ A. Then A^c is an IFGγCS and A^c ⊆ B^c ⊆ γcl(A^c). Therefore B^c is an IFGγCS[5] which implies B is an IFGγOS in X. Hence B ∈ IFGγO(X).

Theorem 3.16: An IFS A of an IFTS (X, τ) is an IFGγOS if and only if F ⊆ γint(A) whenever F is an IFCS and F ⊆ A.

Proof: Necessity: Suppose A is an IFGγOS. Let F be an IFCS, such that F ⊆ A. Then F^c is an IFOS and A^c ⊆ F^c, by hypothesis A^c is an IFGγCS. We have γcl(A^c) ⊆ F^c. Therefore F ⊆ γint(A).

Sufficiency: Let U be an IFOS, such that A^c ⊆ U, and U^c ⊆ A then U^c ⊆ γint(A), by hypothesis. Therefore γcl(A^c) ⊆ U and A^c is an IFGγCS. Hence A is an IFGγOS.

Theorem 3.17: If A is an IFγCS and an IFGγOS in (X, τ). Then A is an IFγOS in (X, τ).

Proof: As A ⊇ A, by hypothesis γint(A) ⊇ A. But we have A ⊇ γint (A). This implies A = γint(A). Hence A is an IFGγOS.

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