Intuitionistic Fuzzy Supra Contra Semi-Continuous Mappings

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Abstract: The purpose of this paper is to introduce a new class of intuitionistic fuzzy supra mappings by using intuitionistic fuzzy supra semi-open sets, known as intuitionistic fuzzy supra contra semi-continuous. We study some of their basic properties. Also, suitable examples, characterizations and relationships among existing mappings have been studied. By means of an example, it is shown that the composition of two intuitionistic fuzzy supra contra semi-continuous functions does not produce an intuitionistic fuzzy supra contra semi-continuous in general. But it can be composed into some other classes of existing intuitionistic fuzzy supra mappings.

Key words and Phrases: IFSTS, IFSSOS, intuitionistic fuzzy semi-supra continuous.

AMS Subject Classification 2010:

1. Introduction

In 1986, the concept of intuitionistic fuzzy set was defined by Atanassov [2] as a generalization of fuzzy set given by Zadeh [9]. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced and investigated the notion of intuitionistic fuzzy topological spaces. In 1987, Abd El-Monsef et al. [1] introduced the fuzzy supra topological spaces and studied fuzzy supra-continuous functions and obtained some properties and characterizations. In 1999, Necla Turan [7] introduced the concept of intuitionistic fuzzy supra topological space. In this paper, we introduce the notion of intuitionistic fuzzy supra contra semi-continuous functions and study their basic properties. Throughout this paper, we will denote the intuitionistic fuzzy supra topological space (briefly, IFSTS) by \((X, \tau)\) or simply \(X\).

2. Preliminaries

We introduce some basic notations and results that are used in the sequel.
(v) \( f^{-1}(B^c) = (f^{-1}(B))^c \);  
(vi) \( (f(A))^c \subseteq f(A^c) \) if \( f \) is surjective. In addition, if \( f \) is injective, then \( (f(A))^c = f(A^c) \).

**Definition 2.4** [3] Let \( X \) and \( Y \) be two non empty sets and \( f : (X, \tau) \rightarrow (Y, \sigma) \) be any function. If \( B = \{ y, \mu_B(y), v_B(y) : y \in Y \} \) is an intuitionistic fuzzy set in \( Y \), then the inverse image of \( B \) under \( f \) is denoted by \( f^{-1}(B) \) is the intuitionistic fuzzy set in \( X \) defined by

\[ f^{-1}(B) = \{ x, \mu_B(f(x)), v_B(f(x)) : x \in X \}, \]

where \( \mu_B(f(x)) = \mu_B(y) \) and \( v_B(f(x)) = v_B(y) \). If \( A = \{ x, \mu_A(y), \nu_A(y) : y \in Y \} \) is an intuitionistic fuzzy set in \( X \), then the image of \( A \) under \( f \) is denoted by \( f(A) \) is the intuitionistic fuzzy set in \( Y \) defined by

\[ f(A) = \{ y, \mu_A(y), 1 - (f(1 - \nu_A))(y) : y \in Y \}. \]

Where,

\[
(f(A))(y) = \begin{cases}  
\sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\
0, & \text{otherwise}
\end{cases}
\]

\[
1 - (f(1 - \nu_A))(y) = \begin{cases}  
\inf_{x \in f^{-1}(y)} \nu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\
1, & \text{otherwise}
\end{cases}
\]

**Definition 2.5** [7] A family \( \tau \) of intuitionistic fuzzy sets on \( X \) is called an intuitionistic fuzzy supra topology (IFSST in short) on \( X \) if \( 0 \in \tau \), \( 1 \in \tau \), and \( \tau \) is closed under arbitrary suprema. Then we call the pair \((X, \tau)\) an intuitionistic fuzzy supra topological space (IFSSTS in short). Each member of \( \tau \) is called an intuitionistic fuzzy supra open set and the complement of an intuitionistic fuzzy supra open set is called an intuitionistic fuzzy supra closed set. The intuitionistic fuzzy supra closure of intuitionistic fuzzy set \( A \) is denoted by \( s - cl(A) \). Here, \( s - cl(A) \) is the intersection of all intuitionistic fuzzy supra closed sets containing \( A \). The intuitionistic fuzzy supra interior of \( A \) will be denoted by \( s - int(A) \). Here, \( s - int(A) \) is the union of all intuitionistic fuzzy supra open sets contained in \( A \).

**Definition 2.6** [8] Let \((X, \tau)\) be an intuitionistic fuzzy supra topological space. An intuitionistic fuzzy set \( A \in IF(X) \) is called

1. intuitionistic fuzzy semi-supra open iff \( A \subseteq s - cl(s - int(A)) \),
2. intuitionistic fuzzy \( \alpha \)-supra open iff \( A \subseteq s - int(s - cl(s - int(A))) \),

The complement of an intuitionistic fuzzy supra open set is called intuitionistic fuzzy supra closed set.

**Definition 2.7** [6] The intuitionistic fuzzy semi-supra interior of a set \( A \) is defined by \( semi - s - int(A) = \bigcup \{ G : G \text{ is an intuitionistic fuzzy semi-supra open set in } X \text{ and } G \subseteq A \} \) and the intuitionistic fuzzy semi-supra closure of a set \( A \) is defined by \( semi - s - cl(A) = \bigcap \{ G : G \text{ is an intuitionistic fuzzy semi-supra closed set in } X \text{ and } G \supseteq A \} \).

**Definition 2.8** [4,6] Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a mapping from an intuitionistic fuzzy supra topological space \( X \) into an intuitionistic fuzzy supra topological space \( Y \). The mapping \( f \) is called an

i) intuitionistic fuzzy contra semi-continuous, if \( f^{-1}(B) \) is an intuitionistic fuzzy semi-closed set in \( X \), for each intuitionistic fuzzy open set \( B \) in \( Y \),

ii) intuitionistic fuzzy semi-supra continuous map if the pre image of each intuitionistic fuzzy supra open set in \( Y \) is an intuitionistic fuzzy super open set in \( X \).

**Theorem 2.9** [6]

(i) Every intuitionistic fuzzy supra open set is intuitionistic fuzzy semi-supra open set.

(ii) Every intuitionistic fuzzy \( \alpha \)-supra open is intuitionistic fuzzy semi-supra open.

**Remark 2.10** [8] \( semi s - int(A) \) is an intuitionistic fuzzy semi-supra open set and \( semi s - cl(A) \) is an intuitionistic fuzzy semi-supra closed set.

**Definition 2.11** Let \((X, \tau)\) and \((Y, \sigma)\) be two intuitionistic fuzzy topological spaces. A map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called intuitionistic fuzzy supra contra continuous map if the inverse image of each intuitionistic fuzzy supra open set in \( Y \) is intuitionistic fuzzy supra closed in \( X \).

**Intuitionistic fuzzy supra contra semi-continuous**

**Definition 3.1** Let \((X, \tau)\) and \((Y, \sigma)\) be intuitionistic fuzzy supra topological spaces. A map...
\( f : (X, \tau) \rightarrow (Y, \sigma) \) is called intuitionistic fuzzy supra contra semi-continuous map if the inverse image of each intuitionistic fuzzy supra open set in \( Y \) is intuitionistic fuzzy semi-supra closed in \( X \).

**Example 3.2**

Let 
\[ X = \{ a, b \}, \quad A = \{ \{ a, 0.2, 0.6 \}, \{ b, 0.3, 0.5 \} \}, \]
\[ B = \{ \{ a, 0.5, 0.4 \}, \{ b, 0.2, 0.6 \} \}, \]
\[ C = \{ \{ a, 0.5, 0.4 \}, \{ b, 0.3, 0.5 \} \} \]
and
\[ \tau = \{ 0, 1 \}, \quad A, B, C \]. Let
\[ Y = \{ p, q \}, \]
\[ D = \{ \{ p, 0.3, 0.4 \}, \{ q, 0.6, 0.2 \} \} \] and
\[ \sigma = \{ 0, 1, D \} \]. Then, \((X, \tau)\) and \((Y, \sigma)\) are intuitionistic fuzzy supra topological spaces. Define a map \( f : (X, \tau) \rightarrow (Y, \sigma) \), by
\[ f(a) = q, \quad f(b) = p \]. Then, \( f \) is an intuitionistic fuzzy supra contra semi-continuous map.

**Theorem 3.3**

A map \( f : (X, \tau) \rightarrow (Y, \sigma) \) is an intuitionistic fuzzy supra contra semi-continuous iff the inverse image of each intuitionistic fuzzy supra closed set in \((Y, \sigma)\) is an intuitionistic fuzzy semi-supra open set in \((X, \tau)\).

**Proof.** It follows from the definition and
\[ f^{-1}(F^c) = (f^{-1}(F))^c \] .

**Theorem 3.4**

Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be a bijective mapping from an intuitionistic fuzzy supra topological space \( X \) into an intuitionistic fuzzy supra topological space \( Y \). If any one of the following properties holds,
1. \( f^{-1}(\text{semi } s - \text{cl}(B)) \subseteq \text{semi } s - \text{int}(\text{semi } s - \text{cl}(f^{-1}(B))) \) for each intuitionistic fuzzy set \( B \) in \((Y, \sigma)\),
2. \( \text{semi } s - \text{cl}(\text{semi } s - \text{int}(f^{-1}(B))) \subseteq f^{-1}(\text{semi } s - \text{int}(B)) \) for each intuitionistic fuzzy set \( B \) in \((Y, \sigma)\),
3. \( f(\text{semi } s - \text{cl}(\text{semi } s - \text{int}(A))) \subseteq \text{semi } s - \text{int}(f(A)) \) for each intuitionistic fuzzy set \( A \) in \((X, \tau)\),
4. \( f(\text{semi } s - \text{cl}(A)) \subseteq \text{semi } s - \text{int}(f(A)) \) for each intuitionistic fuzzy semi-supra open set \( A \) in \((X, \tau)\)

then, \( f \) is an intuitionistic fuzzy supra contra semi-continuous map.

**Proof.** 1 \( \Rightarrow \) 2: It holds by applying 1 for any complement set and then takes complements on both sides.

2 \( \Rightarrow \) 3: Let \( A \) be any intuitionistic fuzzy set in \((X, \tau)\). Let \( B = f(A) \subseteq Y \). Now,
\[ A = f^{-1}(f(A)) \subseteq f^{-1}(B) \]. By hypothesis,
\[ \text{semi } s - \text{cl}(\text{semi } s - \text{int}(f^{-1}(B))) \]
\[ \subseteq f^{-1}(\text{semi } s - \text{int}(B)) \]. Therefore,
\[ f(\text{semi } s - \text{cl}(\text{semi } s - \text{int}(A))) \subseteq \text{semi } s - \text{int}(f(A)) \].

3 \( \Rightarrow \) 4: Let \( B \) be any intuitionistic fuzzy semi-supra open set in \((X, \tau)\). By \([6]\), \( \text{semi } s - \text{int}(A) = A \). By hypothesis,
\[ f(\text{semi } s - \text{cl}(\text{semi } s - \text{int}(A))) \subseteq \text{semi } s - \text{int}(f(A)) \]. Therefore,
\[ f(\text{semi } s - \text{cl}(A)) \subseteq \text{semi } s - \text{int}(f(A)) \].

4 \( \Rightarrow \) intuitionistic fuzzy supra contra semi-continuous: Let \( B \) be any intuitionistic fuzzy supra open set in \((Y, \sigma)\). By \([6]\), \( B \) is an intuitionistic fuzzy semi-supra open set in \((Y, \sigma)\).

Now, \( B = f(A) \) for some
\[ A = f^{-1}(f(A)) \subseteq X \]. By hypothesis,
\[ f(\text{semi } s - \text{cl}(A)) \subseteq \text{semi } s - \text{int}(f(A)) \]. Then,
\[ f(\text{semi } s - \text{cl}(f^{-1}(B))) \subseteq \text{semi } s - \text{int}(f^{-1}(B)). \]
Therefore, \( \text{semi } s - \text{cl}(f^{-1}(B)) \subseteq f^{-1}(B) \).

Always, \( f^{-1}(B) \subseteq \text{semi } s - \text{cl}(f^{-1}(B)). \)

Therefore, \( f^{-1}(B) = \text{semi } s - \text{cl}(f^{-1}(B)) \). By \([6]\), \( f^{-1}(B) \) is intuitionistic fuzzy semi-supra closed in \((X, \tau)\). Hence \( f \) is an intuitionistic fuzzy supra contra semi-continuous map.

**Theorem 3.5**

Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) be any mapping from an intuitionistic fuzzy supra topological space \( X \) into an intuitionistic fuzzy supra topological space \( Y \). Suppose that any one of the following properties hold:
1. \( f(\text{semi } s - \text{cl}(A)) \subseteq \text{semi } s - \text{int}(f(A)) \) for each intuitionistic fuzzy set \( A \) in \((X, \tau)\),
2. \( \text{semi } s - \text{cl}(\text{semi } s - \text{int}(B))) \subseteq f^{-1}(\text{semi } s - \text{int}(B)) \) for each intuitionistic fuzzy set \( B \) in \((Y, \sigma)\),
3. \( f(\text{semi } s - \text{cl}(\text{semi } s - \text{int}(A))) \subseteq \text{semi } s - \text{int}(f(A)) \) for each intuitionistic fuzzy set \( A \) in \((X, \tau)\),
4. \( f(\text{semi } s - \text{cl}(A)) \subseteq \text{semi } s - \text{int}(f(A)) \) for each intuitionistic fuzzy semi-supra open set \( A \) in \((X, \tau)\)

then, \( f \) is an intuitionistic fuzzy supra contra semi-continuous map.

**Proof.** 1 \( \Rightarrow \) 2: It holds by applying 1 for any complement set and then takes complements on both sides.
By hypothesis, 
\[ f^{-1}(B) = A \subseteq X . \]

Thus, 
\[ \text{semi } s - \text{cl}(f^{-1}(B)) \subseteq s - \text{int}(f^{-1}(B)). \]

Hence, 
\[ f^{-1}(A) \text{ is an intuitionistic fuzzy semi-supra closed set in } (Y, \sigma). \]

Remark 3.8. Converse of the above theorem is not true in general as shown in the following example.

Example 3.9. Let 
\[ A = \{\langle a, 0.2, 0.5 \rangle, \langle b, 0.4, 0.3 \rangle\}, \]
\[ B = \{\langle a, 0.5, 0.4 \rangle, \langle b, 0.3, 0.6 \rangle\}, \]
\[ C = \{\langle a, 0.5, 0.4 \rangle, \langle b, 0.4, 0.3 \rangle\} \]

\[ \tau = \{0, 1\} . \]

Let 
\[ f(a) = p, f(b) = q, \]
\[ Y = \{ p, q \} , \]
\[ D = \{ p, 0.4, 0.5 \}, \]
\[ \sigma = \{0, 1\} . \]

\[ (X, \tau) \text{ and } (Y, \sigma) \]

are intuitionistic fuzzy supra topological spaces. A mapping 
\[ f : (X, \tau) \rightarrow (Y, \sigma), \]
\[ f(a) = p, f(b) = q, \]
\[ f(a) = u, f(b) = v, \]
\[ (X, \tau) \text{ and } (Y, \sigma) \]

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\[ \tau = \{0, 1\} . \]

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\[ Y = \{ p, q \} , \]
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\[ f(a) = u, f(b) = v, \]
Let $\tau = \{0, 1, A, B, C\}$. Let $Y = \{p, q\}$, $D = \{p, 0.3, 0.4, q, 0.6, 0.2\}$ and $\sigma = \{0, 1, D\}$. Then, $(X, \tau)$ and $(Y, \sigma)$ are intuitionistic fuzzy supra topological spaces. A mapping $f : (X, \tau) \to (Y, \sigma)$, that is defined by $f(a) = q$, $f(b) = p$, is an intuitionistic fuzzy supra contra semi-continuous mapping. Let $z = \{u, v\}$, $E = \{p, 0.2, 0.6, q, 0.4, 0.3\}$ and $\eta = \{0, 1, E\}$. Then $(Z, \eta)$ is an intuitionistic fuzzy supra topological space. A mapping $g : (Y, \sigma) \to (Z, \eta)$, that is defined by $g(p) = v, g(q) = u$, is an intuitionistic fuzzy supra contra semi-continuous mapping. Define $g \circ f : (X, \tau) \to (Z, \eta)$ by $(g \circ f)(x) = g(f(x))$ for all $x \in X$. Then, it is not an intuitionistic fuzzy supra contra semi-continuous.

**Theorem 3.16** If $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy supra continuous mapping and $g : (Y, \sigma) \to (Z, \eta)$ is intuitionistic fuzzy supra contra continuous mapping, then $g \circ f : (X, \tau) \to (Z, \eta)$ is intuitionistic fuzzy supra contra semi-continuous mapping.

**Proof.** Let $A$ be any intuitionistic fuzzy supra open set in $(Z, \eta)$. By hypothesis, $g^{-1}(A)$ is intuitionistic fuzzy supra closed set in $(Y, \sigma)$. Since $f$ is an intuitionistic fuzzy supra continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is intuitionistic fuzzy supra closed set in $(X, \tau)$. Therefore, $g \circ f$ is an intuitionistic fuzzy supra contra semi-continuous mapping.

**Theorem 3.17** If $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy supra contra semi-continuous mapping and $g : (Y, \sigma) \to (Z, \eta)$ is an intuitionistic fuzzy supra contra semi-continuous mapping, then $g \circ f : (X, \tau) \to (Z, \eta)$ is intuitionistic fuzzy supra contra semi-continuous mapping.

**Proof.** Let $A$ be any intuitionistic fuzzy supra open set in $(Z, \eta)$. Since $g$ is intuitionistic fuzzy supra continuous mapping, $g^{-1}(A)$ is an intuitionistic fuzzy supra open set in $(Y, \sigma)$. Since $f$ is intuitionistic fuzzy supra contra semi-continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is intuitionistic fuzzy supra contra semi-continuous mapping. Therefore, $g \circ f$ is an intuitionistic fuzzy supra contra semi-continuous mapping.

**Theorem 3.18** If $f : (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy supra contra semi-continuous mapping and $g : (Y, \sigma) \to (Z, \eta)$ is an intuitionistic fuzzy supra contra semi-continuous mapping, then $g \circ f : (X, \tau) \to (Z, \eta)$ is an intuitionistic fuzzy supra contra semi-continuous mapping.

**Proof.** Let $A$ be any intuitionistic fuzzy supra open set in $(Z, \eta)$. By hypothesis, $g^{-1}(A)$ is an intuitionistic fuzzy supra closed set in $(Y, \sigma)$. Since $f$ is an intuitionistic fuzzy supra contra semi-continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is intuitionistic fuzzy supra contra semi-continuous mapping. Therefore, $g \circ f$ is an intuitionistic fuzzy supra contra semi-continuous mapping.

4. **References**


