Face-Antimagic Labelings of Subdivided Antiprism

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Abstract. This paper deals with the problem of
labeling the vertices, edges and faces of a plane
graph in such a way that the label of a face and the
labels of vertices and edges surrounding that face
add up to a weight of that face. A labeling of a
plane graph is called \(d\)-antimagic, if for every
positive integer \(z \geq 3\), all \(z\)-sided face weights
constitute an arithmetic progression with a
difference \(d \geq 0\) and starting from a positive
integer \(a' \geq 28\). In this paper, we deal with \(d\)-
antimagic labeling of type \((1,1,1)\) of uniformly
subdivided antiprism and its disjoint copies.

Keywords: \(d\)-antimagic labeling of type \((1,1,1)\),
face-antimagic graph, subdivided
antiprism.

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1 Introduction

In this paper, all graphs are finite, simple,
plane and undirected. A plane graph
\(G = (V,E,F)\) has vertex, edge and face sets,
denoted by \(V(G), E(G)\) and \(F(G)\),
respectively. We denote order of vertex, edge and
face sets as \(|V(G)| = v, |E(G)| = e \) and
\(|F(G)| = f\), respectively.

Labeling of a graph is a map that carries
graph elements to a set of positive integers. If the
domain set consists only on the vertex set of graph
\(G\), then the labeling is called vertex labeling of
type \((1,0,0)\). Similarly, \((0,1,0)\) or \((0,0,1)\)
type labeling is one whose domain carries only
degree or face set, respectively. In \((1,1,0)\) type
labeling, the domain is restricted to vertex and edge
sets.

In this paper, we deal with another case
where the domain of the map is the union of vertex,
edge and face sets. Such a labeling is called face
labeling of type \((1,1,1)\). Mathematically, this
type of labeling is defined as follows:

\[ \lambda: V(G) \cup E(G) \cup F(G) \to \{1,2,\ldots,v+e+f\}. \]

The weight of a face of a graph under the
above labeling \(\lambda\) is the sum of the labels given to
the vertices and edges surrounding that face, and
the face itself. A labeling of type \((1,1,1)\) is called
face-antimagic, if for every positive integer \(z \geq 3\),
all \(z\)-sided face weights of the graph form an
arithmetic progression with a difference \(d \geq 0\)
and starting from a positive integer \(a' \geq 28\). If
\(d = 0\), then such a labeling is called face-magic.

The notion of magic labeling for plane
graphs was explained by Ko-Wei Lih in [11],
where \((1,1,0)\) type magic labeling for prisms,
planar bipyramids and fans in [1], grids in [2],
hexagonal lattices in [3], M"obius ladders in [4],
\(P_n \times P_k\) in [5], prisms in [6] and antiprisms in [7].
P. Jeyanthi and P. Selvagopal in [9] and [12] gave
some new results on magic labeling of type
\((1,1,0)\). Katheresan and Gokulakrishnan proved
\((1,1,1)\) type labeling for the special classes of
plane graphs in [10]. Some more results on such
labelings can be found in [8]. For basic definitions
and notions of graph theory, [13] and [14] can be
seen.

In this paper, we extend this concept to the
graphs which are obtained by subdividing each
edge of the antiprism by equal number of vertices
and we call this a uniform subdivision of antiprism.
In the following sections, we show that subdivided
antiprism and its disjoint copies admit \(d\)-
antimagic labeling of type \((1,1,1)\).

2 The face-antimagic labeling of
graph \(A_{n,1}\)

An \(n\)-sided antiprism \(A_{n}\) is defined as a
polyhedron which is composed of two parallel
copies of some particular \(n\)-sided polygon,
connected by an alternating band of triangles. The
graph \(A_{n,1}\) is obtained by subdividing each edge of
the antiprism \(A_{n}\) by one vertex, as shown in fig. 1.
In the following theorem, we will show that the graph $A_{n_1}$ has $1$-antimagic labeling of type $(1,1,1)$.

**Theorem 1.** For every $n \geq 4$, the graph $A_{n_1}$ admits $(88n + 32, 1)$ face-antimagic labeling of type $(1,1,1)$.

**Proof.**

Let $v = |V(A_{n_1})|$, $e = |E(A_{n_1})|$ and $f = |F(A_{n_1})|$.

We define the vertex, edge and face sets of $A_{n_1}$ as follows:

$$V(A_{n_1}) = \{x_i | 1 \leq i \leq n\} \cup \{y_i | 1 \leq i \leq n\} \cup \{u_i | 1 \leq i \leq n\} \cup \{w_i | 1 \leq i \leq n\} \cup \{v_i | 1 \leq i \leq 2n\}$$

$$E(A_{n_1}) =$$

\[
\{x_i u_i | 1 \leq i \leq n\} \cup \{x_i u_{i+1} | 1 \leq i \leq n-1\} \cup \{x_i w_i | 1 \leq i \leq n\} \cup \{y_i w_i | 1 \leq i \leq n\} \cup \{y_i v_i | 1 \leq i \leq n\} \\
\cup \{y_{2i} v_{2i} | 1 \leq i \leq n\} \cup \{x_i v_{2i} | 1 \leq i \leq n\} \cup \{x_i v_{2i-1} | 1 \leq i \leq n\} \cup \{y_i v_{2i-1} | 1 \leq i \leq n\} \\
\cup \{y_{2i} v_{2i-1} | 1 \leq i \leq n\}
\]

$$F(A_{n_1}) = \{f_{6i} | 1 \leq i \leq 2n\} \cup \{f_{2n,i} | 1 \leq i \leq 2\}$$

Now, we define the labeling $\lambda : V(A_{n_1}) \cup E(A_{n_1}) \cup F(A_{n_1}) \rightarrow \{1, 2, 3, \ldots, v + e + f\}$ as follows:

$$\lambda(x_i) = 2 + i, \text{ for } 1 \leq i \leq n.$$  

$$\lambda(y_i) = \begin{cases} 
3n + i + 2, & \text{for } 2 \leq i \leq n, \\
2n + 3, & \text{for } i = 1.
\end{cases}$$

$$\lambda(u_i) = n + i + 2, \text{ for } 1 \leq i \leq n.$$  

$$\lambda(w_i) = \begin{cases} 
2n + i + 2, & \text{for } 2 \leq i \leq n, \\
3n + 3, & \text{for } i = 1.
\end{cases}$$

$$\lambda(v_i) = 6n - i + 3, \text{ for } 1 \leq i \leq 2n.$$  

$$\lambda(x_i u_i) = 2(8n - i + 2), \text{ for } 1 \leq i \leq n.$$  

$$\lambda(y_i w_i) = 2(7n - i + 1), \text{ for } 1 \leq i \leq n.$$  

$$\lambda(x_i v_{2i}) = 16n - 2i + 3, \text{ for } 1 \leq i \leq n - 1.$$  

$$\lambda(x_i u_{i+1}) = 14n + 3.$$  

$$\lambda(y_{2i} v_{2i-1}) = 2(7n - i + 1), \text{ for } 1 \leq i \leq n - 1.$$  

$$\lambda(y_{2n,i}) = 2(7n + 1).$$  

$$\lambda(x_{2i} v_{2i}) = 10n - 2i + 3, \text{ for } 1 \leq i \leq n.$$
\[ \lambda(y_{i_2}) = 2(3n + i + 1), \quad \text{for } 1 \leq i \leq n. \]
\[ \lambda(x_{i_2}) = 2(5n - i + 1), \quad \text{for } 1 \leq i \leq n - 1. \]
\[ \lambda(y_{i_1}) = 6n + 2i + 1, \quad \text{for } 1 \leq i \leq n. \]
\[ \lambda(x_{i_1}) = 2(5n + 1). \]
\[ \lambda(f_{i_0}) = 10n + i + 2, \quad \text{for } 1 \leq i \leq 2n. \]
\[ \lambda(f_{i_2}) = \begin{cases} 2, & \text{for } i = 1, \\ 1, & \text{for } i = 2. \end{cases} \]

The weights of all \( 2n \) six-sided faces of the graph \( A_{n,1} \) form an arithmetic progression with a difference 1 under the labeling \( \lambda. \)

### 3 The face-antimagic labeling of graph \( mA_{n,1} \)

The graph which is formed by disjoint union of finite number \( m \) of copies of the graph \( A_{n,1} \), is denoted by \( mA_{n,1} \). The next theorem illustrates that the graph \( mA_{n,1} \) admits 1-antimagic labeling of type \( (1,1,1) \).

**Theorem 2.** For every \( n \geq 4 \) and \( m \geq 2 \), the graph \( mA_{n,1} \) admits \((m(8n+12)+20,1)\) face-antimagic labeling of type \((1,1,1)\).

**Proof.**

Let \( v = m \mid V(A_{n,1}) \), \( e = m \mid E(A_{n,1}) \) and \( f = m \mid F(A_{n,1}) \) \( -(m-1) \).

The vertex, edge and face sets of \( mA_{n,1} \) are defined as follows:

\[
V(mA_{n,1}) = \{x_i^k | 1 \leq i \leq n, 1 \leq k \leq m\} \cup \{y_i^k | 1 \leq i \leq n, 1 \leq k \leq m\} \cup \{u_i^k | 1 \leq i \leq n, 1 \leq k \leq m\}
\]
\[
\cup \{w_i^k | 1 \leq i \leq n, 1 \leq k \leq m\} \cup \{v_i^k | 1 \leq i \leq 2n, 1 \leq k \leq m\}
\]

\[
E(mA_{n,1}) = \{x_i^k u_i^k | 1 \leq i \leq n, 1 \leq k \leq m\} \cup \{x_i^k u_{i+1}^{k+1} | 1 \leq i \leq n-1, 1 \leq k \leq m\} \cup \{x_i^k u_{i+1}^{k-1} | 1 \leq i \leq n-2, 1 \leq k \leq m\}
\]
\[
\cup \{y_i^k u_i^k | 1 \leq i \leq n, 1 \leq k \leq m\} \cup \{y_i^k u_{i+1}^{k+1} | 1 \leq i \leq n-1, 1 \leq k \leq m\} \cup \{y_i^k u_{i+1}^{k-1} | 1 \leq i \leq n-2, 1 \leq k \leq m\}
\]
\[
\cup \{x_i^k v_i^k | 1 \leq i \leq n-1, 1 \leq k \leq m\} \cup \{x_i^k v_{i+1}^{k+1} | 1 \leq i \leq n-1, 1 \leq k \leq m\} \cup \{x_i^k v_{i+1}^{k-1} | 1 \leq i \leq n-2, 1 \leq k \leq m\}
\]
\[
\cup \{x_i^k w_i^k | 1 \leq i \leq n-1, 1 \leq k \leq m\} \cup \{x_i^k w_{i+1}^{k+1} | 1 \leq i \leq n-1, 1 \leq k \leq m\} \cup \{x_i^k w_{i+1}^{k-1} | 1 \leq i \leq n-2, 1 \leq k \leq m\}
\]
\[
\cup \{y_i^k w_i^k | 1 \leq i \leq n, 1 \leq k \leq m\} \cup \{y_i^k w_{i+1}^{k+1} | 1 \leq i \leq n-1, 1 \leq k \leq m\} \cup \{y_i^k w_{i+1}^{k-1} | 1 \leq i \leq n-2, 1 \leq k \leq m\}
\]
\[
\cup \{y_i^k v_i^k | 1 \leq i \leq n, 1 \leq k \leq m\} \cup \{y_i^k v_{i+1}^{k+1} | 1 \leq i \leq n-1, 1 \leq k \leq m\} \cup \{y_i^k v_{i+1}^{k-1} | 1 \leq i \leq n-2, 1 \leq k \leq m\}
\]
\[
\cup \{y_i^k f_{i_2} | 1 \leq i \leq 2n, 1 \leq k \leq m\} \cup \{f_{i_2} | 1 \leq i \leq 2n, 1 \leq k \leq m\}
\]

Define a labeling \( \lambda: V(mA_{n,1}) \cup E(mA_{n,1}) \cup F(mA_{n,1}) \rightarrow \{1,2,3,\ldots,v+e+f\} \) as follows:

\[
\lambda(x_i^k) = mi + k + 1, \quad \text{for } 1 \leq i \leq n, 1 \leq k \leq m.
\]
\[
\lambda(y_i^k) = \begin{cases} m(3n+i) + k + 1, & \text{for } 2 \leq i \leq n, 1 \leq k \leq m, \\ m(2n+1) + k + 1, & \text{for } i = 1, 1 \leq k \leq m. \end{cases}
\]
\[
\lambda(u_i^k) = m(n+i) + k + 1, \quad \text{for } 1 \leq i \leq n, 1 \leq k \leq m.
\]
\[
\lambda(w_i^k) = \begin{cases} m(2n+i) + k + 1, & \text{for } 2 \leq i \leq n, 1 \leq k \leq m, \\ m(3n+1) + k + 1, & \text{for } i = 1, 1 \leq k \leq m. \end{cases}
\]
\[
\lambda(v_i^k) = m(6n-i+1) + k + 1, \quad \text{for } 1 \leq i \leq 2n, 1 \leq k \leq m.
\]
\[
\lambda(x_i^k u_i^k) = m(16n+3) - 2(mi - 1) - k, \quad \text{for } 1 \leq i \leq n, 1 \leq k \leq m.
\]
\[
\lambda(x_i^k f_{i_2}) = m(14n + 2) - 2(mi - 1) - k, \quad \text{for } 1 \leq i \leq n, 1 \leq k \leq m.
\]
The labeling $\lambda$ constitutes an arithmetic progression with a difference $1$ as the weights of all $2mn$ six-sided faces of the graph $mA_{n,1}$.

4 The face antimagic labeling of graph $A_{n,k}$

The graph which is obtained by uniformly subdividing each edge of antiprism $A_n$ [11] by $k$ number of vertices, is denoted by $A_{n,k}$. We will prove that the graph $A_{n,k}$ has 1-antimagic labeling of type $(1,1,1)$ in the following theorem.

**Theorem 3.** For every $k \geq 1$, the graph $A_{n,k}$ admits $(16n + 24nk^2 + (48n + 15)k + 17,1)$ face-antimagic labeling of type $(1,1,1)$.

**Proof.**

Let $v = |V(A_{n,k})|$, $e = |E(A_{n,k})|$ and $f = |F(A_{n,k})|$.

The vertex, edge and face sets of $A_{n,k}$ can be defined as follows:

$$V(A_{n,k}) = \{x_i: 1 \leq i \leq n\} \cup \{y_i: 1 \leq i \leq n\} \cup \{u_{i,j}: 1 \leq i \leq n, 1 \leq j \leq k\} \cup \{v_{i,j}: 1 \leq i \leq 2n, 1 \leq j \leq k\} \cup \{w_{i,j}: 1 \leq i \leq n, 1 \leq j \leq k\}$$

$$E(A_{n,k}) = \{x_{u_{i,j}}: 1 \leq i \leq n\} \cup \{u_{i,j}, u_{i+1,j}: 1 \leq i \leq n, 1 \leq j \leq k - 1\} \cup \{x_{u_{i+1,j}}: 1 \leq i \leq n - 1\} \cup \{x_{u_{1,j}}\} \cup \{v_{n,j}, v_{n,j+1}: 1 \leq j \leq 2n, 1 \leq j \leq k - 1\} \cup \{v_{n,j+1}, v_{2n+1-j}: 1 \leq j \leq k - 1\} \cup \{v_{2n+1-j}, v_{2n+1}: 1 \leq j \leq k - 1\} \cup \{v_{2n+1}, v_{2n+2,j}: 1 \leq j \leq n\} \cup \{v_{2n+2,j}, v_{2n+3,j}: 1 \leq j \leq n\} \cup \{x_{v_{2n+3,j}}\} \cup \{y_{i,j}w_{i,j}: 1 \leq i \leq n, 1 \leq j \leq k - 1\} \cup \{w_{i,j}, w_{i+1,j}: 1 \leq i \leq n, 1 \leq j \leq k - 1\} \cup \{w_{i,j}, w_{i+1,j+1}: 1 \leq i \leq n, 1 \leq j \leq k - 1\} \cup \{w_{i+1,j+1}, w_{2n+1-j}: 1 \leq j \leq n\} \cup \{y_{i+1,j+1}, y_{i+1,j+2}: 1 \leq j \leq n\}$$

$$F(A_{n,k}) = \{f_{3(2k+1)+j}: 1 \leq i \leq 2n\} \cup \{f_{4(2k+1)+j}: 1 \leq i \leq 2\}$$

The labeling $\lambda: V(A_{n,k}) \cup E(A_{n,k}) \cup F(A_{n,k}) \to \{1,2,3,...,v+e+f\}$ is defined as follows:

$$\lambda(x_i) = i + 2, \text{ for } 1 \leq i \leq n.$$  
$$\lambda(y_i) = \begin{cases} 3n + i + 2, & \text{for } 2 \leq i \leq n, \\ 2n + 3, & \text{for } i = 1. \end{cases}$$

For $j = 1$. 

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\[ \lambda(u_{i,j}) = n + i + 2, \text{ for } 1 \leq i \leq n. \]

For \( j = 2, 3, \ldots, k \),
\[ \lambda(u_{i,j}) = n(8j + 5) + 3i, \text{ for } 1 \leq i \leq n. \]

For \( j = 1 \),
\[ \lambda(v_{i,j}) = 6n - i + 3, \text{ for } 1 \leq i \leq n. \]

For \( j = 2, 3, \ldots, k \),
\[
\begin{align*}
\lambda(v_{i,j}) &= \begin{cases} 
8n(j+1) + i, & \text{for } i = 1, 2, \\
n(8j + 5) + i + 1, & \text{for } i = 3, 4, \\
n(8j + 5) + i + 2, & \text{for } i = 5, 6, \\
n(8j + 5) + i + (n-1), & \text{for } i = 2n-1, 2n.
\end{cases}
\end{align*}
\]

For \( j = 1 \),
\[ \lambda(w_{i,j}) = \begin{cases} 
2n + i + 2, & \text{for } 2 \leq i \leq n, \\
3n + 3, & \text{for } i = 1.
\end{cases} \]

For \( j = 2, 3, \ldots, k \),
\begin{align*}
\lambda(w_{i,j}) &= 4n(2j + 1) + i + 2, \text{ for } 1 \leq i \leq n. \\
\lambda(x_{u_{i,k}}) &= 2(8n - i + 2), \text{ for } 1 \leq i \leq n. \\
\lambda(x_{u_{i+1,k}}) &= 2(8n - i), \text{ for } 1 \leq i \leq n - 1. \\
\lambda(x_{u_{i,k}}) &= 14n + 3. \\
\lambda(y_{w_{i,1}}) &= 2(7n - i) + 3, \text{ for } 1 \leq i \leq n.
\end{align*}

For \( j = 1, 2, 3, \ldots, k - 1 \),
\begin{align*}
\lambda(w_{i,j}w_{i+1,j}) &= 4n(2j + 3) + i + 2, \text{ for } 1 \leq i \leq n. \\
\lambda(y_{i+1,w_{i,k}}) &= 2(7n - i + 1), \text{ for } 1 \leq i \leq n - 1. \\
\lambda(y_{i,w_{i,k}}) &= 2(7n + 1). \\
\lambda(x_{v_{2i,k}}) &= 2(5n - i + 3), \text{ for } 1 \leq i \leq n. \\
\lambda(x_{v_{2i+1,k}}) &= 2(5n - i - 1), \text{ for } 1 \leq i \leq n - 1. \\
\lambda(x_{v_{2i,k}}) &= 2(5n + 1). \\
\lambda(y_{v_{2i-1,k}}) &= 2(3n + i) + 1, \text{ for } 1 \leq i \leq n. \\
\lambda(y_{v_{2i,k}}) &= 2(3n + i + 1), \text{ for } 1 \leq i \leq n.
\end{align*}

For \( j = 1, 2, 3, \ldots, k - 1 \),
\[ \lambda(u_{i,j}u_{i+1,j}) = n(8j + 11) - 3i + 5, \text{ for } 1 \leq i \leq n. \]

For \( j = 1, 2, 3, \ldots, k - 1 \),
Under the labeling $\lambda$, the weights of all $2n$, $(3k+1)$-sided faces of the graph $A_{n,k}$ constitute an arithmetic progression with a difference 1.

5 The face-magic labeling of graph $A_{8,k}$

The graph which is obtained by uniformly subdividing each edge of the antiprism $A_k$ [11] by $k$ number of vertices, is denoted by $A_{8,k}$. The following theorem shows that the graph $A_{8,k}$ admits face-magic labeling of type $(1,1,1)$.

**Theorem 4.** For every $k \geq 1$, the graph $A_{8,k}$ admits $(a',0)$ face-magic labeling of type $(1,1,1)$ with weight of every $(3k+1)$-sided face equals to $(192k^2 + 399k + 203)$.

**Proof.**

Let $v = |V(A_{8,k})|$, $e = |E(A_{8,k})|$ and $f = |F(A_{8,k})|$. The vertex, edge and face sets of $A_{8,k}$ can be defined as follows:

$V(A_{8,k}) = \{x_i:1 \leq i \leq 8\} \cup \{y_i:1 \leq i \leq 8\} \cup \{u_{i,j}:1 \leq i \leq 8,1 \leq j \leq k\} \cup \{v_{i,j}:1 \leq i \leq 16,1 \leq j \leq k\}$

$E(A_{8,k}) = \{x_{u_{i,j}}:1 \leq i \leq 8\} \cup \{u_{i,j},u_{i,j+1}:1 \leq i \leq 8,1 \leq j \leq k-1\} \cup \{x_{u_{i,1}},u_{i,1}:1 \leq i \leq 8\} \cup \{x_{u_{8,i}},u_{8,i}:1 \leq i \leq 8\}$

$\cup \{y_{v_{h,j+1}}:1 \leq h \leq 16,1 \leq j \leq k-1\} \cup \{y_{v_{2i-1,j}}:1 \leq i \leq 8\} \cup \{y_{v_{2i,j}}:1 \leq i \leq 8\} \cup \{x_{v_{2i,k}}:1 \leq i \leq 8\}$

$\cup \{x_{v_{2i,k}}:1 \leq i \leq 7\} \cup \{x_{y_{v_{h,j+1}}}\cup \{y_{v_{h,j+1}}:1 \leq h \leq 8\} \cup \{w_{v_{i,j}}:1 \leq i \leq 8\} \cup \{w_{v_{i,j+1}}:1 \leq i \leq 8,1 \leq j \leq k-1\}$

$\cup \{y_{w_{h,i}}:1 \leq i \leq 7\} \cup \{y_{w_{h,i}}:1 \leq i \leq 8\}$

$F(A_{8,k}) = \{f_{3(k+1)}:1 \leq i \leq 16\} \cup \{f_{8(k+1)}:1 \leq i \leq 2\}$

Now, the labeling $\lambda:V(A_{8,k}) \cup E(A_{8,k}) \cup F(A_{8,k}) \rightarrow \{1,2,3,...,v+e+f\}$ is defined as follows:

$\lambda(x_i) = 9-i$, for $1 \leq i \leq 8$.

$\lambda(y_i) = \begin{cases} 7+i, & \text{for } 2 \leq i \leq 8, \\ 16, & \text{for } i = 1. \end{cases}$

For $j = 1,2,3,...,k$, $\lambda(u_{i,j}) = (16j+10)+3i$, for $1 \leq i \leq 8$.

For $j = 1,2,3,...,k$, $\lambda(v_{i,j}) = 2(5n+1)+i$, for $1 \leq i \leq 2n$. $\lambda(f_{3(k+1)},i) = \begin{cases} 2, & \text{for } i = 1, \\ 1, & \text{for } i = 2. \end{cases}$
\[ \lambda(v_{i,j}) = \begin{cases} 
64(j+1)+i, & \text{for } i = 1,2, \\
4(16j+10)+i+1, & \text{for } i = 3,4, \\
4(16j+10)+i+2, & \text{for } i = 5,6, \\
4(16j+10)+i+3, & \text{for } i = 7,8, \\
4(16j+10)+i+4, & \text{for } i = 9,10, \\
4(16j+10)+i+5, & \text{for } i = 11,12, \\
4(16j+10)+i+6, & \text{for } i = 13,14, \\
4(16j+10)+i+7, & \text{for } i = 15,16. 
\end{cases} \]

For \( j = 1,2,3,...,k \),

\[ \lambda(w_{i,j}) = 32(2j+1) - i + 3, \text{ for } 1 \leq i \leq 8. \]

\[ \lambda(x_{u_{i,1}}) = \begin{cases} 
i + 25, & \text{for } 1 \leq i \leq 6, \\
33, & \text{for } i = 7, \\
25, & \text{for } i = 8. 
\end{cases} \]

\[ \lambda(x_{u_{i,1}}) = 3(32-i) - 3, \text{ for } 1 \leq i \leq 7. \]

\[ \lambda(x_{u_{i,1}}) = 69, \text{ for } i = 8. \]

\[ \lambda(y_{i,w_{i,1}}) = \begin{cases} 
-(i - 26), & \text{for } 2 \leq i \leq 8, \\
17, & \text{for } i = 1. 
\end{cases} \]

For \( j = 1,2,3,...,k-1 \),

\[ \lambda(w_{i,j}w_{i+1,j}) = 32(2j+3) + i + 2, \text{ for } 1 \leq i \leq 8. \]

\[ \lambda(y_{i+1,w_{i,1}}) = i + 98, \text{ for } 1 \leq i \leq 7. \]

\[ \lambda(y_{i,1,w_{i,1}}) = 106, \text{ for } i = 8. \]

\[ \lambda(x_{v_{2i,1}}) = \begin{cases} 
3(31-i)+1, & \text{for } 2 \leq i \leq 8, \\
67, & \text{for } i = 1. 
\end{cases} \]

\[ \lambda(x_{v_{2i+1,1}}) = 3(30-i)+2, \text{ for } 1 \leq i \leq 7. \]

\[ \lambda(x_{v_{1,1}}) = 68, \text{ for } i = 8. \]

\[ \lambda(y_{i,1,v_{2i-1,1}}) = \begin{cases} 
41, & \text{for } i = 1, \\
-(i - 57), & \text{for } 2 \leq i \leq 4, \\
-(i - 64), & \text{for } 5 \leq i \leq 8. 
\end{cases} \]

\[ \lambda(y_{i,1,v_{2i,1}}) = \begin{cases} 
-(i - 64), & \text{for } 1 \leq i \leq 4, \\
-(i - 57), & \text{for } 5 \leq i \leq 7, \\
64, & \text{for } i = 8. 
\end{cases} \]
For $j = 1, 2, 3, \ldots, k - 1$,

$$\lambda(u_{i,j}u_{i,j+1}) = 4(16j + 22) - 3i + 5, \text{ for } 1 \leq i \leq 8.$$ 

For $j = 1, 2, 3, \ldots, k - 1$,

$$\lambda(v_{h,j}v_{h,j+1}) = \begin{cases} 
64(j + 1) - h + 5, & \text{for } h = 1, 2, \\
4(16j + 22) - h + 4, & \text{for } h = 3, 4, \\
4(16j + 22) - h + 3, & \text{for } h = 5, 6, \\
4(16j + 22) - h + 2, & \text{for } h = 7, 8, \\
4(16j + 22) - h - 1, & \text{for } h = 9, 10, \\
4(16j + 22) - h, & \text{for } h = 11, 12, \\
4(16j + 22) - h - 2, & \text{for } h = 13, 14, \\
4(16j + 22) - h, & \text{for } h = 15, 16. 
\end{cases}$$

$$\lambda(f_{3(k+1)i}) = \begin{cases} 
i + 34, & \text{for } 1 \leq i \leq 6, \text{ & for } 8 \leq i \leq 13, \\
34, & \text{for } i = 7, \\
i + 32, & \text{for } i = 14, \\
i + 33, & \text{for } i = 15, 16. 
\end{cases}$$

$$\lambda(f_{8(k+1)i}) = \begin{cases} 
65, & \text{for } i = 1, \\
66, & \text{for } i = 2. 
\end{cases}$$

All the sixteen $3(k + 1)$ -sided faces of the graph $A_{n,k}$ receive the same weight under the labeling $\lambda$.

6 Conclusion

In this paper, we discussed the face-magic and antimagic labelings of type $(1,1,1)$ for the subdivision of antiprism. We proved that antiprism like graphs, namely $A_{n,1}$, $mA_{n,1}$, $A_{n,k}$ and $A_{8,k}$ admit face-magic and antimagic labelings of type $(1,1,1)$. We end this section by raising the following open problems.

**Open Problem 1.** For $n = 3$, does the graph $A_{n,1}$ admit $d$-antimagic $(\alpha',d)$ labeling of type $(1,1,1)$?

**Open Problem 2.** For every $n \geq 3$, is the graph $A_{n,1}$ $d$-antimagic $(\alpha',d)$ of type $(1,1,1)$, where $d \geq 0$ but $d \neq 1$?

**Open Problem 3.** For every $n \geq 3$, $k \geq 1$, does the graph $A_{n,k}$ admit $d$-antimagic $(\alpha',d)$ labeling of type $(1,1,1)$, where $d \geq 2$?

References


