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Mathematical Modelling and Comparison of Two Degree of Freedom Suspension System of Quarter Car

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Abstract: *The objective of the present study is mainly discussing the mathematical modelling and simulation two degrees of freedom quarter car model. The performance comparison is done between active and passive type suspension system of a passenger car. The performance is then compared to passive suspension, the active suspension provides improved features in terms of riding quality and vehicle handling. The mathematical modelling is developed and the differential equations of motion are derived from it. The equation is derived with the objective of reducing unwanted sprung mass motion such as bounce, pitch, and roll. An Active suspension provides better ride quality and better load distribution on each axle of the car. The stiffness of active suspension can be varied by changing the pneumatic pressure in the pneumatic chamber and this pneumatic pressure can be varied by changing the hydraulic pressure in the hydro-pneumatic chamber. The differential equations are solved with the help of computer simulation using MATLAB. The results include the displacement and acceleration of the sprung and unsprung mass after hitting a bump. It also includes the stability analysis of the system.*

Keywords: *Mathematical modeling, Automobile, Suspension system, MATLAB, Quarter car, Sprung mass*

1. Introduction

The basic concept of land vehicle transportation has not changed much in the last few decades, although much progress was made in improving and optimizing vehicle design and technology. The quest to always go faster further and more comfortably has lead in recent years to the development of advanced suspension systems. An improved suspension system allows a vehicle to achieve higher speeds over rougher terrain and results in better handling, as well as improved ride comfort. In any kind of automobile system, the suspension is required to maintain a grip on the road, to absorb shocks and to dampen vibrations generated by uneven road surface because every road is not perfectly flat and smooth. Even freshly paved highways have subtle imperfections that can affect the wheels of a car. According to newton's laws of motion, all forces have both magnitude and direction. An imperfection on the road on the road causes the wheel to move up and down perpendicularly to the road surface. Without an interfering structure, all of the wheel's energy is transferred to the frame. In such situation, the wheels can lose contact with the road completely and without contact of wheels with the road steering of the vehicle is not possible.

The need of suspension system rose when at the start of 19th century when automobile manufacturing started in mass. Henry Ford with his car Model T and Mercedes-Benz with their engine and powertrain development revolutionized the scenario of an automobile industry. With new technology, new problems aroused which needed to be solved. One of the earliest after developments of automobiles was to design and manufacture a better suspension system for rough roads and pavements. So many companies took that as a challenge and started working on a good suspension system which can give comfort to passengers and improve the ride quality of automobile as well.

A good suspension system is that which absorbs the energy that is generated by the acceleration of vehicle, braking or imperfection on roads without disturbing the upper body of the vehicle. A suspension system must also keep the tires in contact with the road, regardless of road surface. A basic suspension system consists of springs, axles, shock absorbers, arms, rods, and ball joints. Suspension system gives good ride quality and good handling capability to the driver and also it prevents any damage to passengers and goods. Also, it damps the vibration & shocks caused by engine and other moving parts of a vehicle. Also, the ratio of sprung mass and unsprung masses affects the most in the suspension system. The larger the ratio of sprung weight to unsprung weight, the less the body and vehicle occupants are affected by bumps, dips and other road imperfection.

Lu Sun et al. describes walking-beam suspension system traveling at the speed of 20 m/s was used in a case study. The numerical results showed that tires with high air pressure could lead to more damage in pavement structures, and increasing suspension damping and tire damping can reduce the tire loads and pavement damage. Anil Shirahatt et al. describes a case of active suspension system suspension travel increases by 56-60% than passive suspension to provide more ride comfort i.e. less displacement of a sprung mass. Also, it can be concluded since suspension travel and road holding are mutually contradicting parameters, there is an increase in suspension travel in case of active suspension than passive suspension. Ch. Venkateswara Reddy et al. illustrates the mathematical model of a hydro-pneumatic suspension, using Simulink derived the response of a quarter car and studied the effects of road input. A PID controller operates the valve to achieve the desired suspension performance. From the derived results, the acceleration of the sprung mass coming down by 79.5% compared to a traditional system. It also demonstrates that by using hydro-pneumatic suspension model, a reduction in the time length responses to return to static equilibrium positions is achieved, thus improving the performance suspension system and passenger comfort.

Andronic Florin et al. simulated a passive suspension system to verify the accuracy of modeling were used State Space and Transfer Function. Results obtained, using the three methods with the same parameters of the suspension system, are identical. The parameters of a passive suspension system are generally fixed, being chosen to achieve a certain level of compromise between road holding, load carrying, and comfort. Pankaj Sharma et al. analyzed the results of a suspension system for a quarter car model for speed bump of 0.1 m with step input, shows that vehicle sprung mass displacement has to overshoot of 70% and acceleration amplitude of 1.75 m/s^2. The result of unsprung mass displacement also has overshoot of 30% and acceleration drops suddenly from 4 m/s^2 to 0.7 m/s2 which is also undesirable and uncomfortable from the driver's point of view and ride quality. Even though the settling time is quite satisfactory. Sanjeev Chaudhary studies the feasibility of designing interconnected hydro-pneumatic suspensions to be within practically implementable sizes by incorporating parallel mechanical springs to carry a bulk of the vehicle load. The results of the study on interconnected hydro-pneumatic suspension with parallel mechanical springs show that the advantages of the interconnected set up can be retained by compensating the loss of suspension rate through mechanical springs.

2. Suspension System

It is a part of a vehicle that includes spring, shock absorbers, linkages and, steering system including tires & wheels. It connects a vehicle to its wheels and allows relative motion between the two. For any suspension system spring, damper, linkages & pivot points are necessary. A spring is an element that stores the energy when a vehicle is hit to bump or pit. It absorbs the energy and releases afterwards, for a pothole, spring release energy, and stores afterwards. A damper is a device that deadens, restrains, or depresses. Without a damper, car spring will extend and release energy an uncontrollable rate. Linkages are used for transmitting movements and pivot points are used for support linkages.

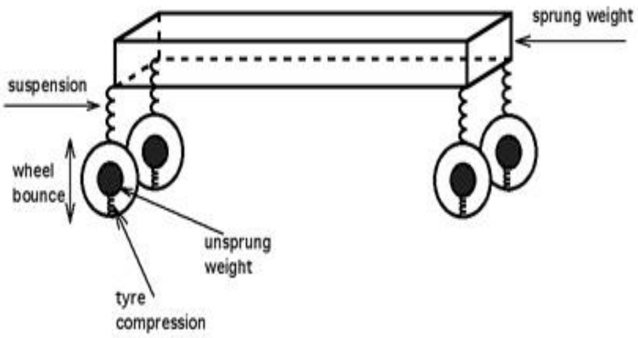

Fig. 2.1 Suspension system setup

There are two types of a suspension system is considered for this research work. Active type suspension and Passive type suspension. The passive suspension system is known as conventional suspension system which is widely used over the years and still very popular. The reasons for popularity of the system are low cost and ease of manufacturing. The passive suspension system is considered as an open loop system that means it does not have any control unit or any feedback element. So there are no chances of controlled operation of this type of system in some cases, which sometimes leads to catastrophic events.

Active suspension systems are considered as modern suspension system with the advanced control unit integrated with it. In active suspension system suspension rate of spring can be changed any time and damping control is also possible. It can be further divided into two types Hydro-pneumatic & Electromagnetic. Hydraulically actuated suspensions are controlled with the help of servo motor mechanisms while in electromagnetic suspension system it uses linear electromagnetic motors. Ride Quality is defined as the degree of comfort and protection provided by the vehicle when vehicle runs on the different type of surface like smooth road surface, terrain or the off-road. A vehicle which gets disturbed with minor road irregularities has low ride quality. Low ride quality produces unwanted vibrations and noises which reduce the life of the vehicle. It also causes discomfort to passengers and fatigue on a long journey. For a good ride quality vehicle must resist unwanted pitch, roll, and bounce. These very much depend on the suspension system of the vehicle.

3. Problem Formulation

3.1 Mathematical Modelling of the System

In present work, two degrees of freedom system is modelled as a problem statement. The system includes two mass viz. sprung mass and unsprung mass. The system model can be expressed as:

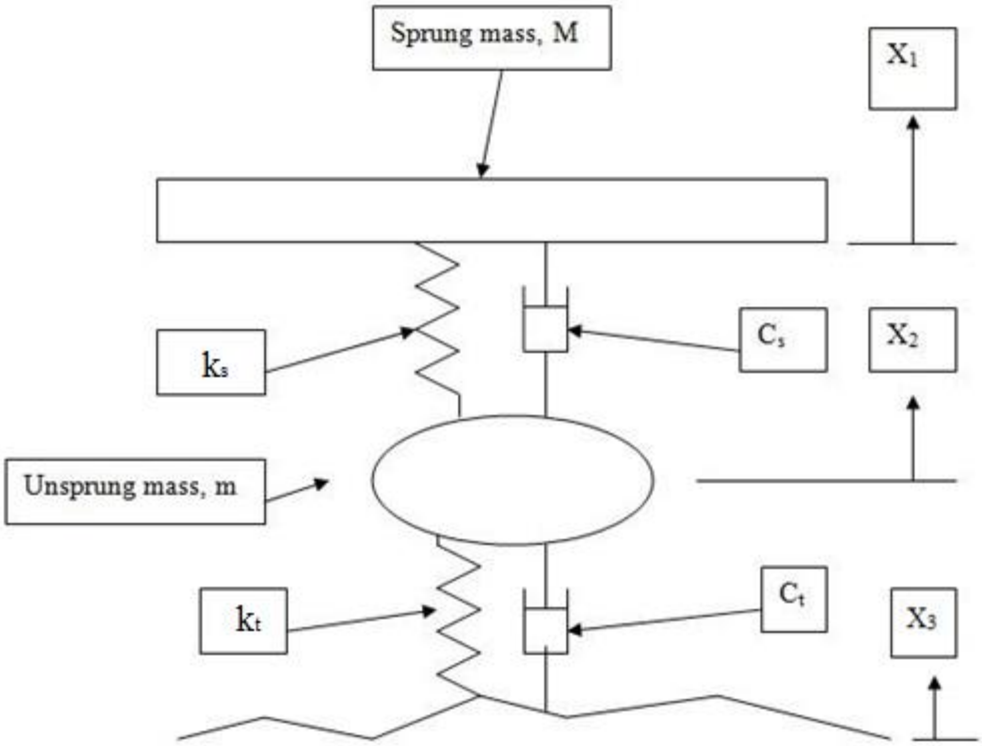

Fig. 3.1 Mathematical model of 2-DOF system

A sprung mass is defined by M; unsprung mass is defined by m. Both tire and suspension is assumed to have damping effect and both possess fix values of stiffness. Here linear characteristics of spring and damper are assumed, while formulating the model of the system. The system is linearly dependent on time and assumed to have no non-linear effects.

3.2 Derivation of Governing Equations of the system

After formulating the mathematical model of the system, now the governing equations need to be derived to determine the behavior of the system. These equations are in form of differential equation of second degree, because of 2-DOF of the model. For derivation first, the model is divided into two parts. One part includes the sprung mass and the other part includes the unsprung mass. For simplification in deriving the equations, Free Body Diagram is presented with depicts the forces that are acting on the system.

3.2.1 Sprung Mass

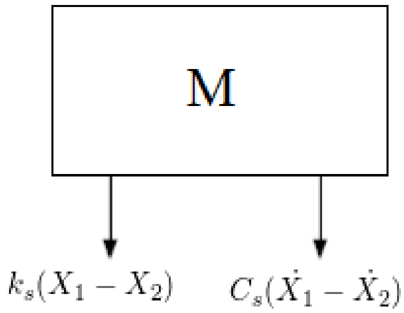

Fig. 3.2 Free Body Diagram-1

The forces which are acting on the sprung mass, when vehicle hits to a bump, is presented in the above diagram. The sign of downward forces is taken as positive and for upward forces it is negative. In the case of sprung mass two downward forces act in general. The forces are generally assumed to be time invariant and the effect of gravity is neglected.

Using Newton's second law of motion,

$$M\ddot{x}_1 + k_s(x_1 - x_2) + C_s(\dot{x}_1 - \dot{x}_2) = 0$$

__________________ (1)

3.2.2 Unsprung Mass

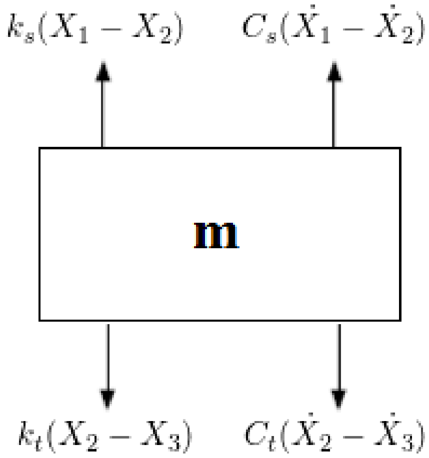

Fig. 3.3 Free Body Diagram-2

The forces which are acting on the unsprung mass, are depicted in the figure. For this condition, vehicle is hitting a bump. Four different forces act on unsprung mass due to four different elements attached to it. Using this free body diagram, we can determine the governing differential equation.

Using Newton's second law of motion,

$$m\ddot{x} + k_t(x_2 - x_3) + C_t(\dot{x}_2 - \dot{x}_3) - k_s(x_1 - x_2) - C_s(\dot{x}_1 - \dot{x}_2) = 0$$

__________ (2)

3.3 Derivation of Transfer Function of the system

From Equation (1) and (2), Taking Laplace Transform of equation (1) and (2),

$Ms^2X_1(s) + k_sX_1(s) – k_sX_2(s) + C_sSX_1(s) – C_sSX_2(s) = 0$ ____ (3)
Also,
$mS^2X_2(s) + k_tX_2(s) – k_tX_3(s) + C_tSX_2(s) – C_tSX_3(s) – k_sX_1(s) + k_sX_2(s) – C_sSX_1(s) + C_sSX_2(s) = 0$ _________ (4)

From these equations,

$[\ MS^2 + k_s + C_sS\ ]\ X_1(s) – [\ k_s + C_sS\ ]\ X_2(S) = 0$ ____(5)
And,
$[\ mS^2 + k_t + C_tS – k_s – C_sS\ ]\ X_1(s) + [\ k_t + C_tS + k_s + C_sS\ ]\ X_2(s) + [–k_t – C_tS\ ]\ X_3(s) = 0$ _________________(6)

Converting equations (5) and (6) into matrix form,

$$\begin{bmatrix} (MS^2 + Ks + CsS) & -(ks + CsS) \\ (mS^2 + kt + CtS - ks - CsS) & (kt + CtS + ks + CsS) \end{bmatrix} \begin{bmatrix} X1(s) \\ X2(s) \end{bmatrix} + \begin{bmatrix} 0 \\ -(kt + CtS) \end{bmatrix} X_3(s) = 0$$
_____________________ (7)

3.3.1 Unsprung Mass Displacement

To determine displacement of sprung mass with respect to time, a classic Cramer's technique is used to solve the equation (7).

Applying Cramer's rule for expression (7),

$$\frac{X_2\ (S)}{X_3\ (S)} = \frac{\begin{vmatrix} (MS^2+ks+CsS) & 0 \\ (MS^2+ks+CsS) & -(kt+CtS) \end{vmatrix}}{\begin{vmatrix} (MS^2+ks+CsS) & -(ks+CsS) \\ (MS^2+ks+CsS) & (kt+CtS+ks+CsS) \end{vmatrix}}$$

So,

$$\frac{X_2\ (S)}{X_3\ (S)} = \frac{-\ (kt+CtS)\ (MS^2+Ks+CsS)}{[(MS^2+Ks+CsS)\ (kt+CtS-ks-CsS)] + [(mS^2+kt+CtS-ks-CsS)\ (ks+CsS)]}$$

$$\frac{X_2\ (S)}{X_3\ (S)} = \frac{[MS^2kt+kskt+CsSkt+MS^3Ct+CtSks+CsCtS^2]}{\begin{matrix} MS^2kt+MS^3\,CT+MS^2ks+MS^3Cs+\text{kskt}+\text{SCtks}+\text{k}^2s+SksCs+SCskt+S^2CsCt+SCsks \\ +S^2Cs^2+S^2mks+S^3mCs+ktks+ktCsS+SCtks+S^2CsCt-\text{k}^2s-SCsks-SCsks-S^2Cs^2 \end{matrix}}$$

$$\frac{X_2\ (S)}{X_3\ (S)} = \frac{-[(MCt)S^3+(MkT+CsCt\ )S^2+(CtksCskt)S+kskt]}{(MCt+MCs+mCs)S^3+(Mks+Mkt+mks+\ 2CsCt\ )S^2+(2CtCs+2Cskt)S+(2ktks)}$$ ____________(8)

Here, the power of numerator is 3 and, the power of denominator is also 3. For transfer function analysis the power of denominator must be greater than or equal to the power of numerator. So, in this case, this transfer function can be solved. These differential equations can be solved by MATLAB with proper numerical inputs. This will give the exact results instead of solving by numerical methods.

3.3.2 Sprung Mass Displacement

Similarly, here, to determine the displacement of unsprung mass, first it is required to derive the transfer function. For this equation (7) is used as input constrain. Here displacement of unsprung mass is derived with respect to road irregularities.

Applying Cramer's rule for expression (7),

$$\frac{X_1(S)}{X_3(S)} = \frac{\begin{vmatrix} 0 & -(kt+CsS) \\ -(kt+CtS)) & kt+Cts+ks+csS \end{vmatrix}}{\begin{vmatrix} (MS^2+ks+CsS) & -(ks+CsS) \\ (mS^2+ks+CtS-ks-CsS) & (kt+CtS+ks+CsS) \end{vmatrix}}$$

$$\frac{X_1(S)}{X_3(S)} = \frac{-(ks+CsS)(kt+CtS)}{\begin{vmatrix} (MS^2+ks+CsS) & -(ks+CsS) \\ (mS^2+ks+CtS-ks-CsS) & (kt+CtS+ks+CsS) \end{vmatrix}}$$

$$\frac{X_1(S)}{X_3(S)} = \frac{kskt+ksCtS+ktCsS+S^2}{(MCt+MCs+mCs)S^3+(Mks+Mkt+mks+2CsCt)S^2+(2CtCs+2Cskt)S+(2ktks)}$$

$$\frac{X_1(S)}{X_3(S)} = \frac{-[(CsCt)S^2+(ksCt+ktCs)S+kskt]}{(MCt+MCs+mCs)S^3+(Mks+Mkt+mks+2CsCt)S^2+(2CtCs+2Cskt)S+(2ktks)} \qquad (9)$$

4. Results and analysis

Table 4.1 Numerical input data

No.	Parameters	Passive Suspension System Values	Active Suspension System Values
1	Sprung Mass, M	400 Kg	400 kg
2	Unsprung Mass, m	30 Kg	30 kg
3	Suspension Spring Stiffness, k_s	15000 N/m	25000 N/m
4	Suspension Damping Co-efficient, Cs	1120 N-s/m	1000 N-s/m
5	Tire Stiffness, k_t	310000 N/m	310000 N/m
6	Tire Damping Co-efficient, C_t	3100 N-s/m	3100 N-s/m
7	Speed Bump Height, X_3	10 cm	10 cm

Graphical results of active and passive type suspension system are done using a quarter car suspension model. For this first, a MATLAB code is used to solve the transfer function. A bump of 10 cm height is assumed as an input of the system. The input signal is assumed to be a step input which varies with the time. MATLAB results show the displacement of sprung mass and unsprung mass with respect to the pavement surface. From the displacement and settling time, we get the actual behavior of the system. Also, peak overshoot is helpful is predicting the stability of the vehicle.

4.1 Passive Suspension System Analysis

Taking the data mentioned in the figure as system constraints, following results were determined.

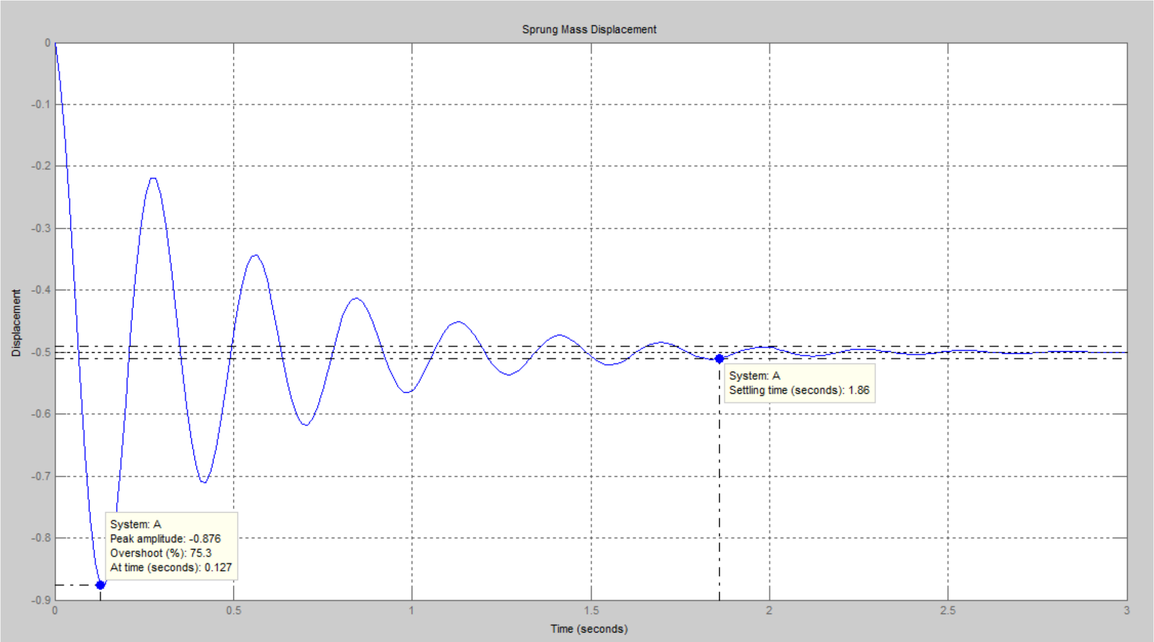

Fig. 4.1 Sprung mass displacement for Passive Suspension System

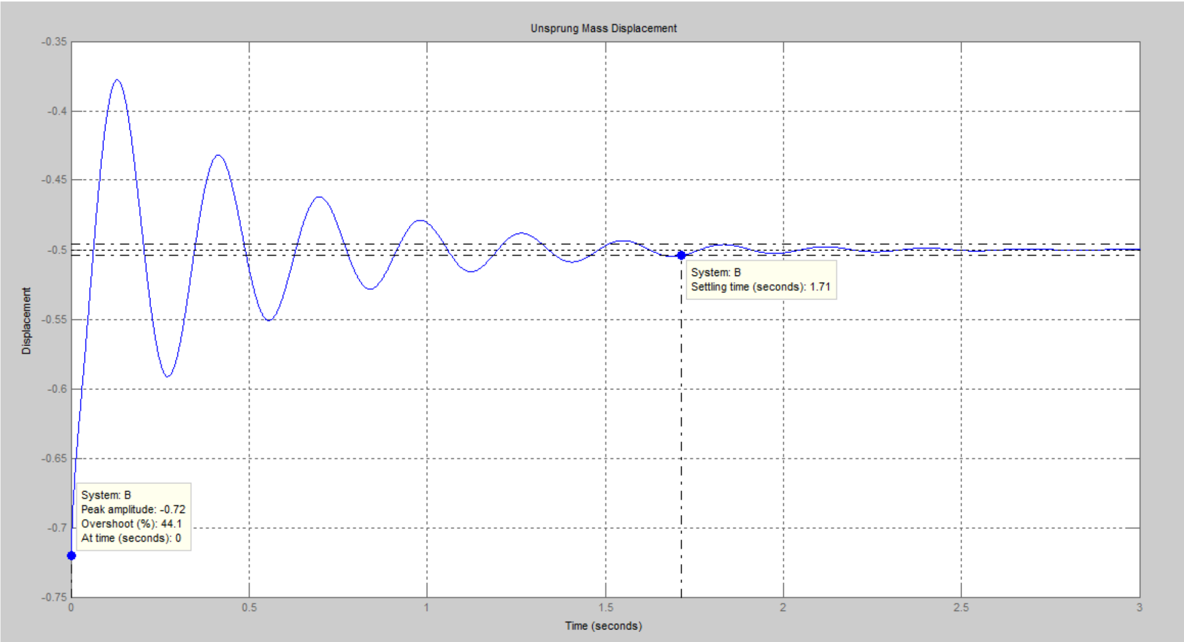

Fig. 4.2 Unsprung mass displacement for Passive Suspension System

From the Fig.5.1 it illustrates that the settling time of the sprung mass for passive suspension system is 1.86 seconds, and for the unsprung mass it is 1.71 seconds. Pick overshoot for the sprung mass is 75.3% at 0.127 seconds and for the unsprung mass, pick overshoot is 44.1% at 0 second. Both sprung mass and unsprung mass are settled at a displacement of -0.5m after the settling time is achieved.

4.2 Active Suspension System Analysis

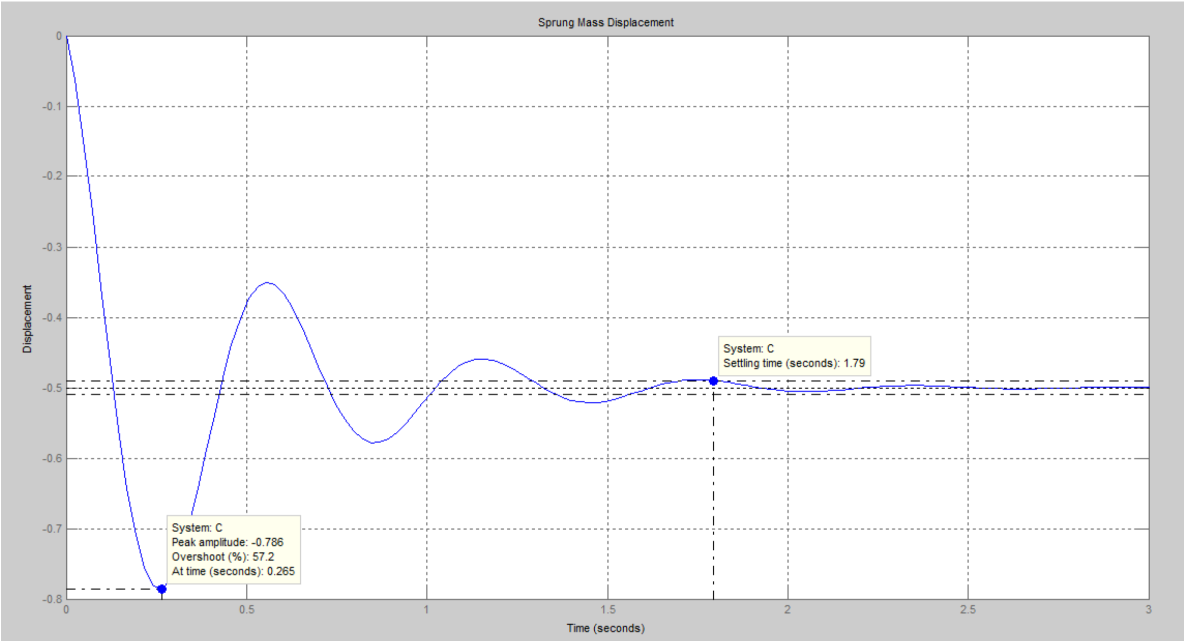

Fig. 4.3 Sprung Mass displacement for Active Suspension System

For an Active Suspension System, the sprung mass displacement with respect to time has shown. The settling time of the system is found to be 1.79 seconds and the pick overshoot is found to be 57.2% at 0.265 seconds. From the graph, we can say that the fluctuations of mass with respect to time are less compared to that of passive suspension. This shows that the active type suspension system can aid in adjusting more compared to other suspension systems.

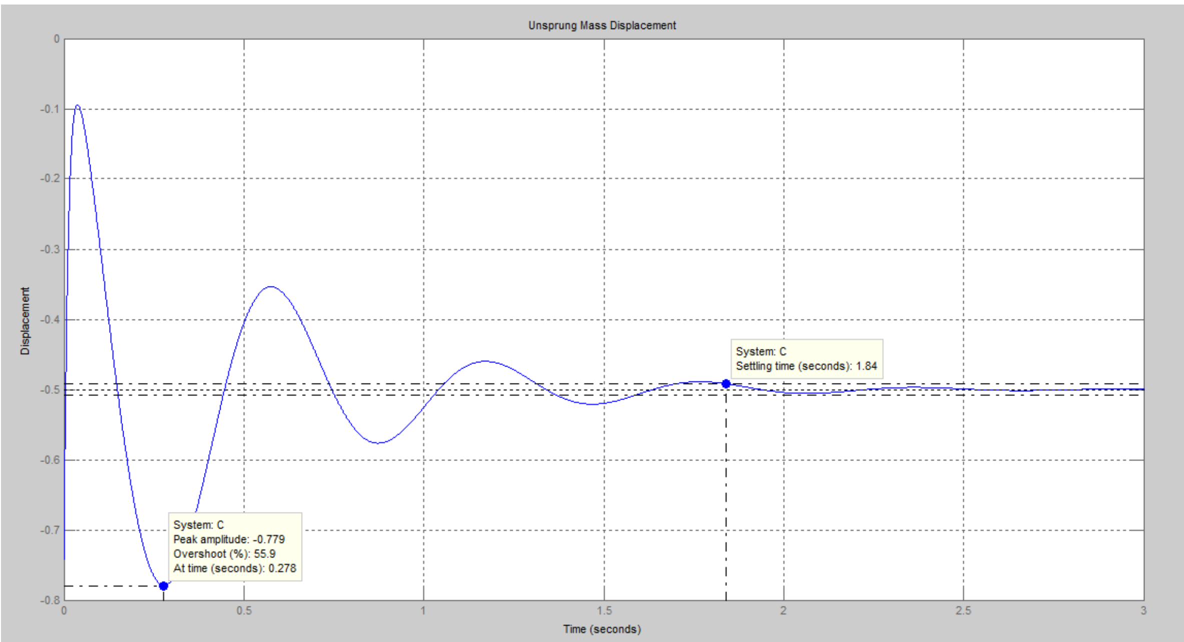

Fig. 4.4 Unsprung Mass displacement for Active Suspension System

Similarly, as sprung mass, the results of unsprung mass displacement for an active suspension system is derived. For unsprung mass in the active type suspension system, the settling time for the system is found to be 1.84 seconds and the pick overshoot is found to be 55.9% at approximately 0.278 seconds. Here the settling time of unsprung mass is estimated more in the active suspension system compared to the passive suspension system.

From the above results, it can be said that the overall settling time of sprung mass is found less in the overall settling time of sprung mass is found less in the case of sprung mass displacement of Active of type Suspension System. This result shows that, whenever vehicle gets hit to bump or pit, the active type suspension aids in settling in less time compared to a passive suspension. This is very much helpful particularly in rough pavements, where chances of comfortless driving are high. Also, this result shows that passengers will experience fewer shocks and will feel comfortable even in the bumpy ride.

In the case of a passive suspension system, a high value of overshoot is undesirable in vehicle's performance. High overshoot is also undesirable for better working and performance of suspension and for its long life. High values of overshoot tend to bring drastic changes in acceleration of vehicle which is not good for comfort point of view.

5. Conclusion

The work of this project concludes the superiority of the active type suspension over the passive type suspension. The results prove that the reliability and comfort quality of the active type suspension is better. Also, the work shows that ride quality in the active suspension system is proven to be better than other suspension systems. Also using closed loop suspension system, we can have a more compatible system for transportation. Countries like India has very rough pavements and roads in especially in villages, so the use of this suspension can make a big different in public and commercial transportation. So the data that is derived can be used in the further improvement of the system.

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