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Blow-Up Behavior for a Quasilinear Parabolic Equation with Nonlinear Boundary Condition

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***Abstract**: We study the solution of an initial boundary value problem for a quasilinear parabolic equation with a non-linear boundary condition. We first show that any positive solution blows-up in finite time. Then we study the global and non-global existence of positive solutions of a non-linear parabolic equation. Finally we also construct some self -similar single point blow-up patterns with different oscillations*

***Keywords** : quasilinear parabolic equation, non-linear boundary condition, blow-up, self similar, Laypunov function.*

1. INTRODUCTION

Fluid dynamics is essentially the study of gases and liquids in motion. It involves calculating various properties of a particular fluid such as its velocity, pressure, temperature and density.

Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, determining the mass flow rate of petroleum through pipelines, predicating weather patterns and reportedly modelling fission weapon detonation. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid. we established ‘ solution of an initial boundary value problem for a quasilinear parabolic equation with a nonlinear boundary conditions ’.

2. PRELIMINARIES

Definition 2.1

Fluid dynamics is essentialy the study of gases and liquids in motion. It involves calculating various properties of a particular fluid such as its velocity, pressure, temperature and density

Definition 2.2

A first order partial differential equation is said to be a **quasi linear equation.** If it is linear in p and q

(i.e) if it is of the form

$P(x, y, z)\, p + Q\, (x, y, z)\, q = R(x, y, z)$

Example:

$$(x^2 + z^2)p - xyq = z^3x + y^2$$

3. BLOW-UP BEHAVIOR FOR A QUASILINEAR PARABOLIC EQUATION WITH NONLINEAR BOUNDARY CONDITION

3.1 Theorem

Suppose that $q > 0$. Then for everypositive bounded smooth initial data u_0, there exists a finite time $T > 0$ such that

$$\lim_{t\to T^-} sup\left\{\max_{x\epsilon[0,1]} u(x,t)\right\} = \infty$$

Proof

Given

Suppose that $q > 0$. Then for every positive bounded smooth initial data u_0, there exists a finite time $T > 0$

To prove :

$$\lim_{t\to T^-} sup\left\{\max_{x\epsilon[0,1]} u(x,t)\right\} = \infty$$

By assumption, there is a positive constant δ such that $u_0 \geq \delta$ in $[0,1]$.

Then, by the maximum principle, $u(x,t) \geq \delta$ for the corresponding solution u of (P).

We introduce the following quantity

$$N(t) := \int_0^1 u^{-\gamma}(x,t)\, dx$$

By differentiating $N(t)$ and using

$$u_t = u^{1+\gamma} u_{xx},\ 0 < x < 1, t > 0,$$

$$u_x(0,t) = -u^q(0,t)\ \ u_x(1,t) = 0, t > 0$$

We get,

$$N'(t) = -\gamma u^q(0,t)$$

Since $q > 0$, We get

$$N' \leq -\eta$$

for some constant $\eta > 0$.

Thus $N(t)$ should vanish at some finite time.Therefore, the solution u cannot be bounded for all $t > 0$.

This implies that there exists a finite $T < 0$, such that (1) holds and the theorem is proved.

Hence the proof

3.2 Theorem

Suppose that $q > 1$. Under the assumption $u_0' \leq 0$, $u_0'' \geq 0$ in [0,1], $x = 0$ is the only blow-up point.

Proof

Given

Suppose that $q > 1$.

Under the assumption $u_0' \leq 0$, $u_0'' \geq 0$ in [0,1].

To prove :

$x = 0$ is the only blow - up point.

Suppose, for contradiction, that there exists another blow-up point $a \in (0,1]$

Then any point $b \in [0,1]$ is also a blow-up point,

Since $u_x < 0$ and $u_t > 0$.

Now we fix any number $b \in (0,1)$

we consider the function

$$J(x,t) := u_x(x,t) + \varepsilon h u^q(x,t),$$

$$h(x) := (x-b)^2,\ \varepsilon > 0.$$

Then it is easy to compute that

$$J_t - u^{1+\gamma} J_{xx} - (1+\gamma)\, u^\gamma u_x =$$
$$-\varepsilon\,(1+q) q h u^{\gamma+q-1} u_x^2$$
$$-\varepsilon(1+\gamma+2q) h' u^{\gamma+q} u_x - \varepsilon h'' u^{\gamma+q+1}$$
$$\leq 0$$

by using the properties of h and the fact that $u_x < 0$.

Clearly, $J(b,t) < 0$ for all $t \in (0,T)$.

More over,

$J(0,t) = -u^q(0,t)(1 - \varepsilon b^2) \leq 0$ for all

$t \in (0,T)$,

If, $\varepsilon < 1/b^2$

By choosing ε small enough and using $u_x(x, T/2) < 0$ in $[0,b]$,

we have $J(x, T/2) \leq 0$ for all $x \in [0,b]$.

Therefore, it follows from the maximum principle that $J \leq 0$ in $[0.b] \times [T/2, T)$

i.e., $-u^{-q}(x,t)\, u_x(x,t) \geq \varepsilon\,(x-b)^2$,

$x \in [0,b]\ \ t \in [T/2, T)$

Integrating the above equation from 0 to b,

We get

$$[u^{1-q} - u^{1-q}(0,t)]/\ (q-1)$$
$$\geq \varepsilon \int_0^b (x-b)^2\, dx$$
$$= \varepsilon\, b^2/3\ \ \forall\, t \in (T/2, T).$$

Let $t \uparrow T^-$, we reach a contradiction.

Hence the proof

4. GLOBAL AND NON-GLOBAL SOLUTIONS OF A NONLINEAR PARABOLIC EQUATION

4.1 Theorem

There holds $\phi(\xi) \to 0$ as $\xi \to \infty$.

Proof

Since $\phi(\xi)$ is monotone decreasing, the limit

$$l = \lim_{\xi \to \infty} \phi(\xi)$$

Exists and $l \geq 0$. Suppose > 0 ,

We define,

$$H(\xi) = \frac{1}{2}[\phi'(\xi)]^2 + G\left(\phi(\xi)\right), \quad \xi > 0$$

Where,

$$G(\phi) = \int_\eta^\phi (s^p + \alpha s^{1-\sigma})\, ds\ , \ l \le \phi < \eta.$$

Since

$$H'(\xi) = -\left(\frac{n-1}{\xi} + \beta\xi\phi^{-\sigma}\right)\phi'(\xi) \ge 0$$

And $\qquad G(\phi) \ge -(\eta^p + \alpha l^{1-\sigma})\ (\eta - l)$

Exists and $L > -\infty$. by the definition of $H(\xi)$ the limit

$$-K = \lim_{\xi\to\infty} \phi'(\xi)$$

Exists and $K \ge 0$.

since

$$\int_0^\infty \phi'(\xi)\ d\xi = l - \eta,$$

there exists a sequence $\{\xi_k\}$ such that $\xi_k \to 0$ as $k \to \infty$.

Hence $K = 0$.

Therefore , there exists $M > 0$ such that $|\phi(\xi)| \le M$ and $|\phi'(\xi)| \le M,\ \forall\, \xi \ge 0$ dividing $\phi'' + \frac{n-1}{\xi}\phi' + \phi^p + \alpha\phi^{1-\sigma} + \beta\xi\phi^{-\sigma}\phi' = 0$ by ξ,

we have

$$\frac{\phi''(\xi)}{\xi} + \frac{n-1}{\xi^2}\ \phi'(\xi)$$
$$= -\frac{\phi^p(\xi)}{\xi} - \alpha\frac{\phi^{1-\sigma}}{\xi}\beta\phi^{-\sigma}(\xi)\phi'(\xi)$$

Integrating it from 1 to ξ_k, we obtain

$$\left|\int_1^{\xi_k}\frac{\phi''(\xi)}{\xi} + \frac{n-1}{\xi^2}\ \phi'(\xi)\right| = \left|\frac{\phi'(\xi_k)}{\xi_k} - \phi'(1) + \int_1^{\xi_k}\frac{n}{\xi^2}\ \phi'(\xi)\right|$$

$$\le M + |\phi'(1)| + nM$$

$$\left|\int_1^{\xi_k}\beta\phi^{-\sigma}(\xi)\phi'(\xi)\right| = \left|\frac{\beta}{1-\sigma}\left(\phi^{1-\sigma}(\xi_k) - \phi^{1-\sigma}(1)\right)\right|$$

$$\le \frac{\beta}{\sigma-1}\left(l^{1-\sigma} + \phi^{1-\sigma}(1)\right)$$

But,

$$\int_1^{\xi_k}\frac{\phi^p(\xi) + \alpha\phi^{1-\sigma}(\xi)}{\xi} \ge \alpha\, l^{1-\sigma}\int_1^{\xi_k}\frac{1}{\xi} \to \infty$$

as $k \to \infty$, a contradiction

Hence, $l = 0$.

Hence the proof

CONCLUSION

In this dissertation we discussed the solution of an initial boundary value problem for a quasilinear parabolic equation with a non-linear boundary conditions. We first shown that any positive solution blows up in finite time. Then constructing a Lyapunov function , we have proved the convergence of the solution. Finally we have discussed the global and non-global existence of positive solutions of a non-linear parabolic equations.

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